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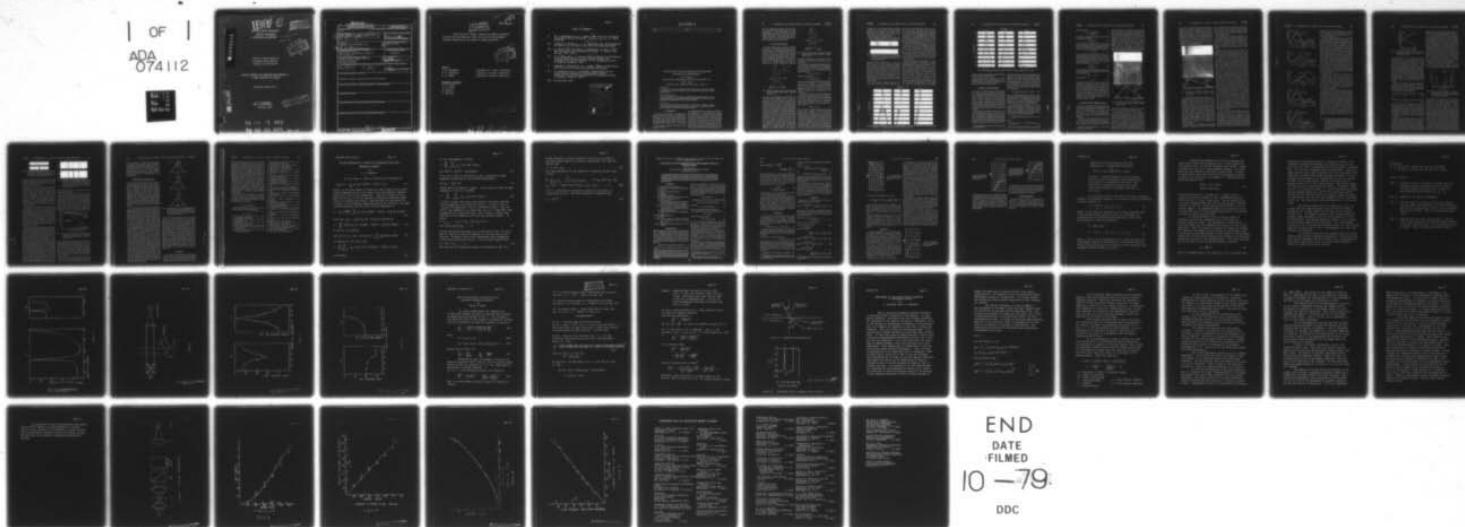
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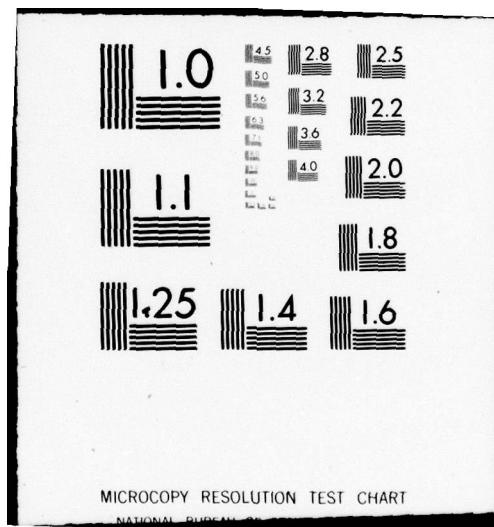
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OPTICAL METHODS FOR ABSOLUTE MEASUREMENT OF  
SOUND PRESSURE IN LIQUIDS.

Technical Report No. 5.

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E. A. Hiedemann  
Main Investigator  
February 1962

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## THE STUDY OF ULTRASONIC WAVEFORM BY OPTICAL METHODS \*

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### Summary

A discussion is given of the experimental and theoretical aspects of the optical methods for studying the waveform of ultrasonic waves in liquids. The relative merits of these methods are given.

### Zusammenfassung

Die experimentellen und theoretischen Grundlagen der optischen Methoden zur Untersuchung der Wellenform von Ultraschallwellen in Flüssigkeiten werden erörtert. Die speziellen Vorteile dieser Methoden werden miteinander verglichen.

### Sommaire

On discute au double point de vue théorique et expérimental les différentes méthodes optiques employées pour étudier la forme d'une onde ultra-sonore se propageant dans un liquide. On compare les mérites individuels de ces méthodes.

### 1. Introduction

The study of the waveform of ultrasonic waves travelling in liquids has increased in significance in the past few years since there has been both direct and indirect evidence that waves become distorted when progressing in these liquids. Studies of the waveforms of these distorted waves are important

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to an understanding of the processes which lead to this distortion. In a recent publication, ZAREMBO and KRASIL'NIKOV [1] reviewed research concerning this "finite amplitude distortion". Optical methods have played an important role in the experimental investigation of this distortion, since these methods appear to be both sensitive and accurate, and have the advantage that the measuring device does not disturb the sound field. An attempt will be made here to discuss optical methods for determining wave-

form, to compare these methods, to show how they are related and to discuss possibilities for further measurements. The theoretical behavior of finite amplitude distortion will not be described here. For further information on that subject the reader is referred to the review by ZAREMBO and KRASIL'NIKOV and to the references given in that review.

## 2. General observations

The optical methods used to study the waveform of ultrasonic waves utilize the diffraction by plane ultrasonic waves of collimated light incident normal to the ultrasonic waves. In these methods, the assumption is made that the light is phase modulated as it passes through the ultrasonic waves. This behavior is illustrated schematically in Figs. 1 and 2. Fig. 1 indicates this behavior for a sinusoidal ultrasonic wave. A collimated light beam is shown traveling in the  $z$ -direction through the ultrasonic

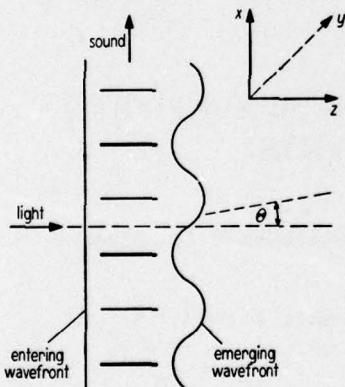


Fig. 1. Schematic diagram of the wavefronts of light before and after passing through a sinusoidal ultrasonic wave.

wave which is progressing in the  $x$ -direction. The wavefront of the impinging light is a plane represented by the vertical line in the figure and the ultrasonic wavefronts are planes represented by the short horizontal lines, the heavy lines representing condensations and the light ones representing rarefactions. The light passing through the condensations is retarded relative to that passing through the rarefactions. If the light is not bent significantly as it passes through the ultrasonic beam, the resulting wavefront is corrugated and has the same form as the ultrasonic wave, i.e. the light is phase modulated. For a sinusoidal ultrasonic wave the modulation is sinusoidal; for a distorted ultrasonic wave, the resulting light wavefront contains a corresponding distortion. As an example, Fig. 2 illustrates the behavior of a triangular ultrasonic wave. The amplitudes of the corrugations shown in these figures

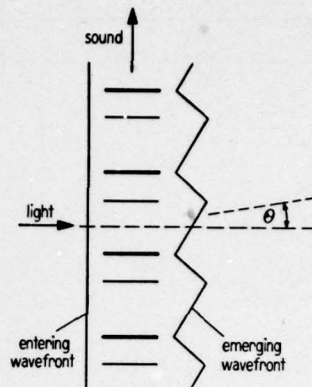


Fig. 2. Schematic diagram of the wavefronts of light before and after passing through a triangular ultrasonic wave.

are greatly exaggerated, since the changes in index of refraction produced by the ultrasonic waves are quite small.

The condition for considering the resulting wavefront to be phase modulated is discussed mathematically by EXTERMANN and WANNIER [2] and MIKHAILOV and SHUTILOV [3]. For sinusoidal waves this condition becomes

$$2\pi^2 L^2 \mu' / \lambda^{*2} \mu_0 \ll 1, \quad (1)$$

and for the triangular wave

$$4 L^2 \mu' / b^2 \mu_0 \ll 1, \quad (2)$$

where  $L$  is the path length of light through the ultrasonic wave of wavelength  $\lambda^*$ ,  $\mu'$  is the maximum change of index of refraction produced by the ultrasonic wave,  $\mu_0$  the index of refraction of the undisturbed medium and  $b$  is the shortest distance between a rarefaction and condensation for a triangular wave.

In practice the light wave has finite dimensions. A dimension of particular interest is its width  $D$  in the  $x$ -direction. RAMAN and NATH [4] predicted that, for periodic ultrasonic waves, if  $D$  is very much greater than the fundamental sound wave length  $\lambda^*$ , then the light is diffracted at discrete angles  $\theta$  (see Fig. 1) given by

$$\sin \theta = -n \lambda / \lambda^*, \quad (3)$$

where  $\lambda$  is the wave length of the light and  $n$  is any integer positive, negative or zero. The intensity distribution in the diffraction orders for sinusoidal ultrasonic waves is also given by this theory. It can be seen from the development in Section 3 that the separate diffraction orders can be resolved for a  $D$  of several wavelengths. The assumption that the waveform is periodic restricts eq. (3) and the following discussion to ultrasonic waves which do not change noticeably over the width of the light beam  $D$ .  $D$  can usually be chosen so that this condition

is satisfied for waves undergoing finite amplitude distortion. If  $D$  is less than  $\lambda^*$ , distinct orders of diffraction are not obtained and the intensity distribution over  $\theta$  is continuous. Such a pattern is commonly referred to as a broadened image.

Fig. 3 illustrates this behavior for wide and narrow beams of light. Fig. 3 a is a photograph of a



Fig. 3. Photographs of typical ultrasonic diffraction patterns observed using (a) a wide light beam and (b) a narrow light beam. HARGROVE, ZANKEL and HIEDEMANN [15].

diffraction pattern obtained when a wide beam of light was used. (A one Mc/s progressive ultrasonic wave travelling in water produced this pattern.) Fig. 3 b is a photograph taken of a diffraction pattern produced by the same ultrasonic wave when the light beam width was limited to one quarter of a sound wave length. In general, the patterns produced using a narrow beam of light appear to be blurred images of the discrete diffraction patterns which would have resulted from the use of a wide beam of light.

For normal incidence of light, sinusoidal ultrasonic waves produce symmetrical diffraction pat-

terns. Since distorted ultrasonic waves may not contain the symmetry of sinusoidal waves, one might expect that resulting diffraction patterns would not be symmetrical in  $\theta$ . Although an earlier suggestion was made by LOEBER and HIEDEMANN [5] that the asymmetry in a broadened image could be related to finite amplitude distortion of an ultrasonic wave, BREAZEAL and HIEDEMANN [6], [7] were the first to successfully relate such asymmetries to wave distortion. They showed that this asymmetry increased as the distortion increased, i. e. with increased ultrasonic pressure and propagation distance. Patterns of this type which were recently obtained by BREAZEAL [8] are shown in Fig. 4. These pictures clearly indicate that the broadened images, which were produced by progressive waves, become more asymmetrical when the distance and/or sound pressure is increased.

The first quantitative measurements of finite amplitude distortion by optical methods were made independently by MIKHAILOV and SHUTILOV [3], [9], [10] and ZANKEL and HIEDEMANN [11] to [13]. Both of these experiments utilized the study of discrete diffraction patterns, but the methods of interpreting these patterns were quite different. Pictures of this type of diffraction, which were made by BREAZEAL [8], are shown in Fig. 5. A picture similar to this was given by MIKHAILOV and SHUTILOV [9]. The pictures in Figs. 4 and 5 were taken under identical experimental conditions except for the width of the light beam  $D$ . It should be noted again that the diffraction patterns obtained

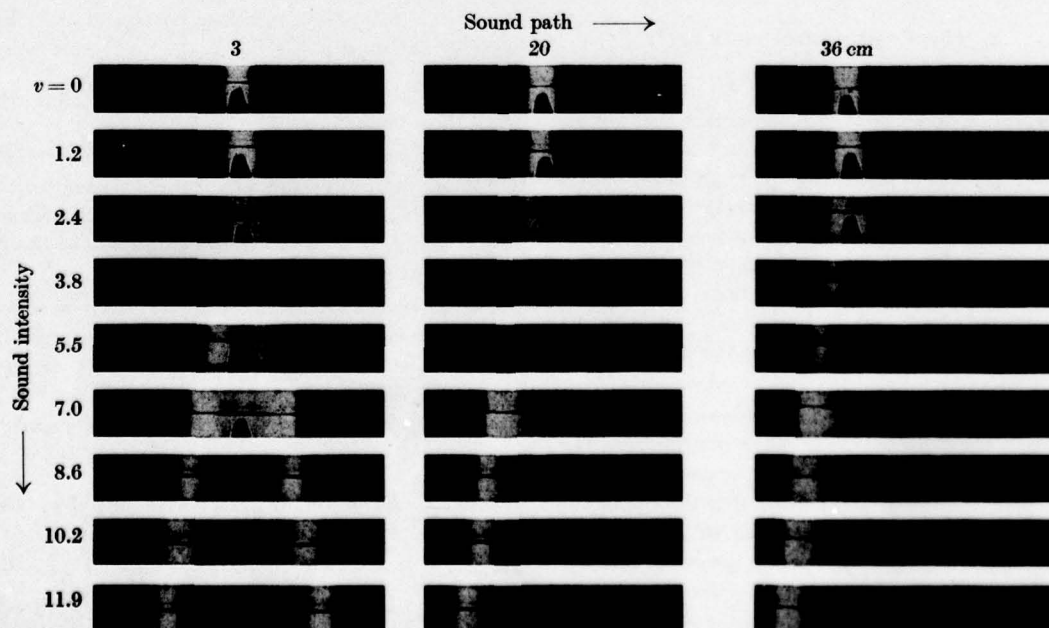


Fig. 4. Photographs of diffraction of a narrow beam of light,  $D = \lambda^*/2$ , by a 1,76 Mc/s ultrasonic wave progressing in water for various sound intensities and distances. BREAZEAL [8].

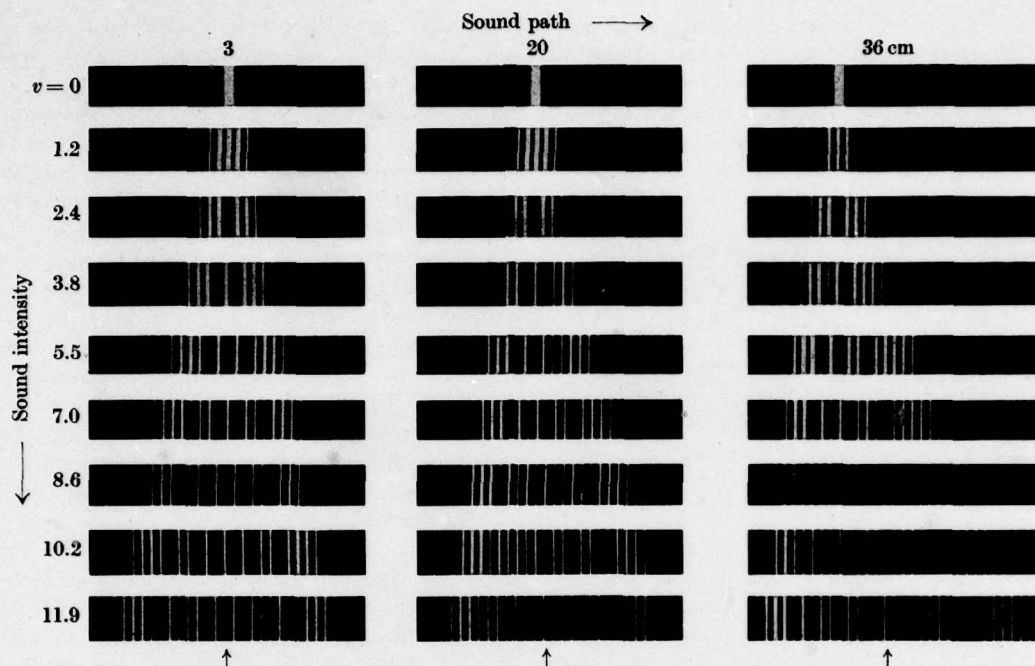


Fig. 5. Photographs similar to those of Fig. 4 using a wide beam of light. BREAZEALE [8].

using the narrow beam appear to be blurred images of those obtained using wide beams.

Other optical methods which have since been developed will be discussed following a more detailed explanation of the methods of MIKHAILOV and SHUTILOV and of ZANKEL and HIEDEMANN.

### 3. Theory of optical methods<sup>1</sup>

In order to correlate the observed diffraction effects and waveform in greater detail it is necessary to extend the RAMAN-NATH theory to include the case of arbitrary waveform and arbitrary beam width  $D$ . Since the studies of waveform have been mainly confined to progressive waves, stationary waves will not be discussed further in this paper. General principles which apply to these methods will be discussed here and each method will later be treated as a specific application of the more general principles.

Assuming that the ultrasonic waveform does not change in the region  $D$ , the wave may be resolved into its fundamental and higher harmonic components. The additional assumption that the change of index of refraction is proportional to the instantaneous change of pressure, leads to the result that the emerging wavefront has a form which can be re-

solved into the same FOURIER components as the ultrasonic wave, with a constant of proportionality between the two which can be determined experimentally or predicted theoretically. For distorted finite amplitude waves, it has been predicted that the odd harmonics are in phase and the even harmonics are  $180^\circ$  out of phase with the fundamental. The change of index of refraction could, for this case, be written as

$$\Delta\mu = \sum_{j=1}^{\infty} a_j \mu \sin 2\pi j[\nu^* t - (x/\lambda^*)], \quad (4)$$

where  $\mu$  is the maximum change of index of refraction produced by the fundamental component of the ultrasonic wave,  $\nu^*$  is the frequency of the fundamental and  $a_j$  is the ratio of the amplitude of the  $j^{\text{th}}$  harmonic component to that of the fundamental.

The diffraction pattern resulting from a wavefront produced by such an ultrasonic wave may be obtained by integrating the contributions of each portion of the wavefront in the interval  $D$ . For a distorted finite amplitude wave the light intensity as a function of  $t$  at a particular angle  $\theta$  is

$$I(t) = \sum_{n,m=-\infty}^{\infty} \Phi_n \Phi_m W_n W_m \cos 2\pi(n-m)\nu^* t, \quad (5)$$

where

$$\Phi_n = \sum_{k_1, k_2, k_3, \dots, k=-\infty}^{\infty} J_{n-2k_1-3k_2-\dots}(v) J_{k_1}(a_2 v) J_{k_2}(a_3 v) \dots \quad (6)$$

$J_n(v)$  is the  $n^{\text{th}}$  order BESSEL function of argument  $v$ ,

<sup>1</sup> The equations given here are taken from [11], [13], [14] and [15].

and  $W_n$  is given by

$$W_n = \frac{\sin \pi G(H+n)}{\pi G(H+n)}. \quad (7)$$

$G$  is the ratio of light beam width to sound wavelength and  $H$  is defined by

$$\sin \theta = H \lambda / \lambda^*. \quad (8)$$

It is seen that  $H$  is the deflection in terms of separation of diffraction orders were they present and, since  $\theta$  is very small,  $H$  is approximately proportional to  $\theta$ . The RAMAN-NATH parameter  $v$  is given by

$$v = 2 \pi \mu L / \lambda. \quad (9)$$

As was indicated previously, these results assume that each harmonic is either in phase or  $180^\circ$  out of phase with the fundamental. For arbitrary phase, cosine terms must be considered in the FOURIER series description of  $\Delta \mu$ . The time average light intensity is found from eq. (5) to be

$$I = \sum_{n=-\infty}^{\infty} \Phi_n^2 W_n^2. \quad (10)$$

For  $G \gg 1$ , that is, wide light beams, the light intensity is independent of time and gives discrete orders at angles  $\theta$  given by

$$\sin \theta = -n \lambda / \lambda^*, \quad (3)$$

and of intensity<sup>2</sup>

$$I_n = \Phi_n^2. \quad (11)$$

This special case applies to the study of ultrasonic waveform by observation of discrete orders of diffraction. For sinusoidal waves,  $\Phi_n$  reduces to the BESSEL Function predicted by RAMAN and NATH. It is seen that the broadened image predicted by eq. (10) can be thought of as being produced by the discrete orders predicted by eq. (11) which are broadened by the weighting function  $W_n^2$ . This produces the blurring which was referred to in the discussion of Figs. 3, 4 and 5.

#### 4. Studies of discrete diffraction spectra

Studies of waveform using discrete diffraction spectra produced by distorted finite amplitude waves were made independently by MIKHAILOV and SHUTILOV [3], [9], [10] and ZANKEL and HIEDEMANN [11] to [13]. Since the interpretation of their results differed, their methods will be treated separately.

The sound waves used by MIKHAILOV and SHUTILOV were of considerably higher intensity than those used by ZANKEL and HIEDEMANN. Using ultrasonic waves producing a large number of orders, they

<sup>2</sup> This result for two superposed waves was first obtained by MERTENS [16].

observed that there is a position of maximum intensity on each side of the diffraction spectrum. For sinusoidal waves these maxima are symmetric about the zero order. For example, the orders of maximum intensity in Fig. 3 are plus and minus five. For distorted waves these maxima do not occur symmetrically, i.e. the maxima do not occur at equal distances from the zero order and one maximum is brighter than the other. This is illustrated in Fig. 6 a. Fig. 6 b is a photogram of this image. In some cases the dim maximum cannot be distinguished as a maximum, as is illustrated by the pat-

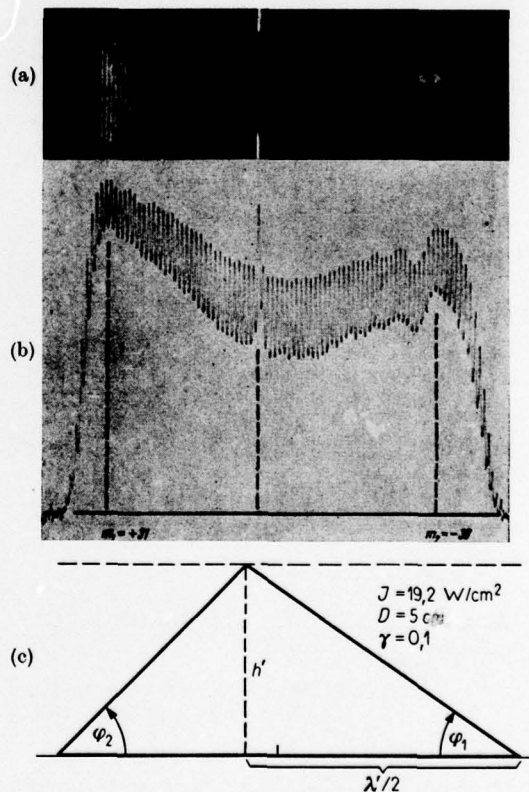


Fig. 6. Construction of the relative waveform from optical data. Both intensity maxima visible. SHUTILOV [10].

tern in Fig. 7 a. MIKHAILOV and SHUTILOV used the positions of maxima in interpreting their results.

For sinusoidal waves, and a large number of orders, the angular position  $\theta_m$  of a maximum is approximately proportional to the maximum absolute value of gradient of sound pressure. A sinusoidal wave may be approximated by a symmetric triangular wave, with slopes equal to the maximum sound pressure gradients. To a good approximation the  $\theta_m$ 's predicted for the sinusoidal wave would be the same as those predicted for the triangular wave. MIKHAILOV and SHUTILOV suggested that a distorted

wave could be approximated by an asymmetric triangular wave. The two positions  $\theta_m$  of maximum intensity could then be determined from the two slopes of the triangle, with the constant of proportionality being the same as that experimentally determined for sinusoidal waves. Triangular models of the ultrasonic waveforms which produced the diffraction patterns are given in Figs. 6 c and 7 c. A possibility for the actual form of the wave is illustrated by the dashed line in Fig. 7 c.

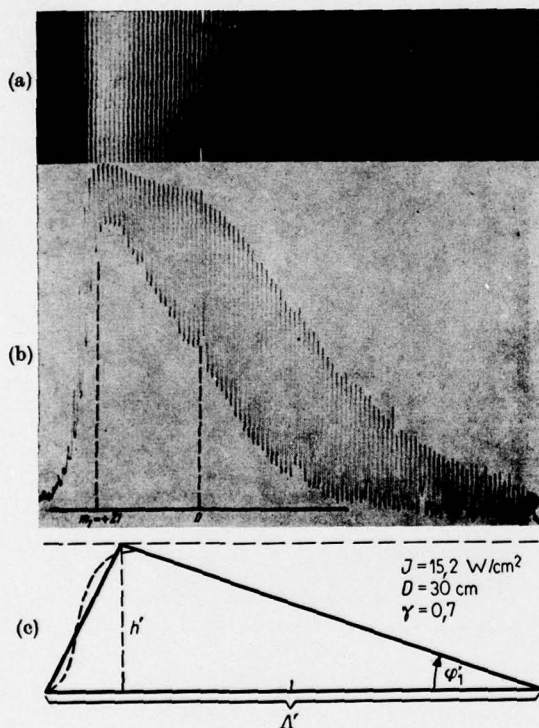


Fig. 7. Construction of the relative waveform from optical data. One intensity maximum visible. SHUTILOV [10].

To obtain further results by this method, additional information was needed. This information was furnished by knowledge of the behavior of finite amplitude distortion. MIKHAILOV and SHUTILOV assumed that the crests of the ultrasonic waves moved at a constant rate which was faster than that of the troughs. The model thus obtained is that of a triangle, originally isosceles, which approaches a saw-tooth in form as it progresses. These studies were made under conditions of low ultrasonic absorption, and the assumption was made that absorption effects could be neglected. This research led to determinations of distortion and a distortion parameter for water which were in reasonable agreement with theory.

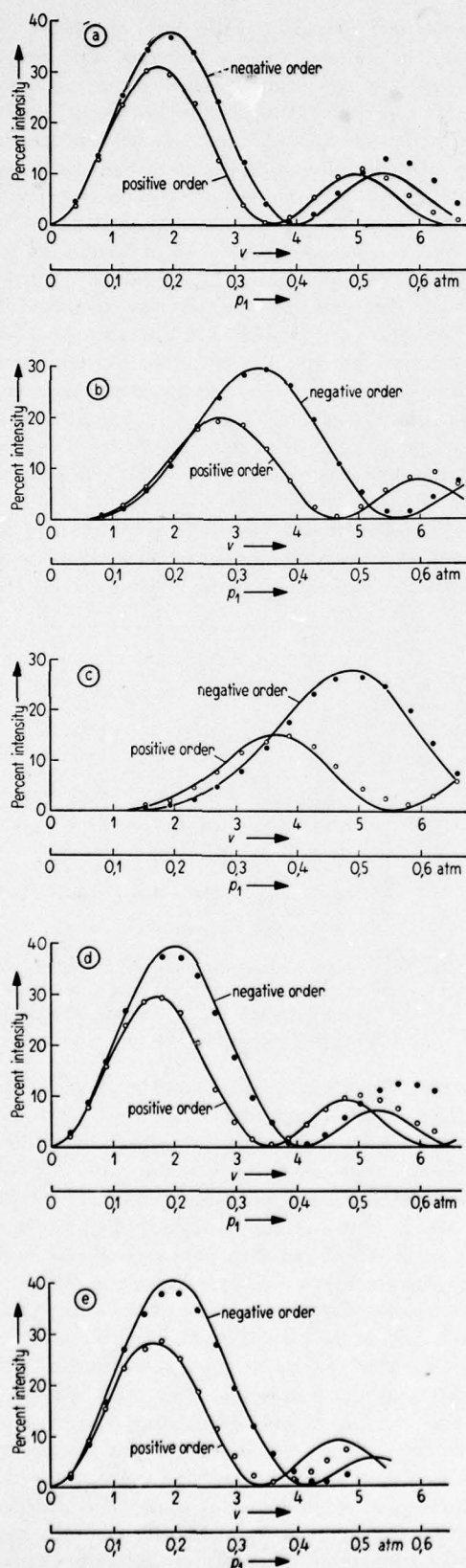
Further evidence of the validity of this method was given. To determine a measure of distortion the

$\theta_m$  on one side of the zero order only is needed. The  $\theta_m$  on the other side was used as a check of consistency. There was also agreement between theory and measurements of distortion at different sound pressures and distances.

The simplicity of the method recommends its use in obtaining information about the maximum absolute values of the pressure gradient. The method is obviously less useful for studies of detailed waveform. If more detailed information about the distortion is desired, one should use other methods which may require, however, more tedious calculations.

Clearly, more information could be obtained by studying the relative light intensity in all of the diffraction orders, rather than by simply determining  $\theta_m$ . In order to do this the triangular approximation can no longer be used. One may resort instead to a comparison of eq. (11) with experiment. ZANKEL and HIEDEMANN used this method of approach to study finite amplitude distortion. They measured the intensity of light in the first three positive and first three negative diffraction orders. Since their studies were confined to relatively low sound intensities where few orders were observable, most of the information was contained in these orders. In addition, they also relied on theories on finite amplitude distortion to furnish additional information in order to simplify comparisons. The method consisted in measuring the intensities of the diffracted orders. These values were then compared with those obtained from eq. (11) by varying the distortion parameter until the calculated intensities agreed with those measured. The distortion and the distortion parameter obtained for carbon tetrachloride were in reasonable agreement with theory. Several examples of the comparison between theory and experiment are shown in Fig. 8. These figures are plots of theoretical (lines) and experimental (dots) light intensities as a function of fundamental ultrasonic pressure. The curves are for various distances and orders using a single distortion parameter for all the curves. Deviations between theory and experiment can be explained by approximations made in the treatment of the prediction of the finite amplitude distortion, which become less valid at higher ultrasonic intensities.

As has been mentioned, this method has the advantage that it yields more detailed information about the wave form. Also one need not be concerned about the effect of absorption, and indeed this method was used under conditions in which absorption was very significant. The disadvantages of this method are that the calculations are tedious, and that the method of varying parameters until agreement is obtained may not always be practical.



(The latter was not significant for the work of ZANKEL and HIEDEMANN since interpretation of their results required only one parameter which is a property of the liquid used.)

A further difference between the two methods should be mentioned. Eq. (11) assumes that the average sound intensity is constant throughout the cross section of the light beam. It also assumes that the waveform does not change significantly over the light beam width  $D$ . In most cases the structures of the sound beam is noticeable. The observed diffraction pattern can then be thought of as being the sum of patterns produced by sections of the beam which are homogeneous. This tends to remove some of the detailed structure necessary for complete utilization of eq. (11). To remove this difficulty, ZANKEL and HIEDEMANN limited the light beam dimensions so that the beam passed through portions of the sound wave which could be considered nearly homogeneous. SHUTILOV observed that his diffraction patterns sometimes contained localized minima near  $\theta_m$ . BREAZEALE [8] has shown that these minima become quite pronounced when the light beam width is limited (see Fig. 5).

It appears that for the method of evaluation used by MIKHAILOV and SHUTILOV a large number of orders and a large amount of asymmetry are necessary. Their method is thus valid for high ultrasonic pressures, and their work was limited to such a region. The more detailed method of evaluation does not contain such a limitation and was, therefore, found useful at relatively low amplitudes in the presence of low distortion.

It has been mentioned that the procedure of varying parameters used by ZANKEL and HIEDEMANN may not be practical for all cases. A method of quickly determining the second harmonic content of distorted waves has recently been suggested by HARGROVE [17]. He observed that if all harmonics higher than the second are neglected and that if the phase between the harmonic and fundamental is either zero or  $180^\circ$ , then the average value of the intensities of the first orders calculated from eq. (11) is independent of  $a_2$  (the percent of second harmonic) when  $v = 2.40$ . He also observed that there are simple, almost linear, relationships between the intensities of each of these first orders and  $a_2$ . This

Fig. 8. The light intensity of various diffraction orders as a function of ultrasonic pressure. Points are experimental; lines theoretical: (a) first orders at 3 Mc/s and 7 cm, (b) second orders at 3 Mc/s and 7 cm, (c) third orders at 3 Mc/s and 7 cm, (d) first orders at 3 Mc/s and 14 cm, and (e) first orders at 3 Mc/s and 21 cm. ZANKEL and HIEDEMANN [11].

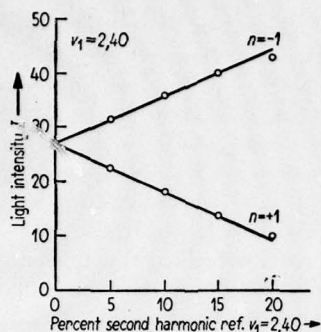


Fig. 9. Calculated light intensity in the first orders as a function of percent second harmonic for  $v = 2.40$ . HARGROVE [17].

is shown in Fig. 9. It is seen that the intensity in each of these orders is quite sensitive to the amount of second harmonic present. His procedure is as follows. First the intensity at the transducer is varied until  $I_1 + I_{-1} = 0.542$ . This corresponds to  $v = 2.40$ . Then, with the knowledge of  $I_1$  and  $I_{-1}$ , the percentage of second harmonic is read directly from Fig. 9. It is recognized that this method works at only one value of  $v$  and that it assumes no harmonics higher than the second. However, this method is relatively unexplored and it may be possible to find criteria for other values of  $v$ , which might include higher harmonics, and which would make this method more flexible.

Another direct method of interpreting diffraction results has been suggested by Cook [18], [19]. This method is more involved than that suggested by HARGROVE, but is more complete. Cook found that, under the conditions previously stated,

$$\cos(2\pi \Delta\mu L/\lambda) = \sum_{n=-\infty}^{\infty} \Phi_n \cos\{2\pi n[v^*t - (x/\lambda^*)]\} \quad (12)$$

and

$$\sin(2\pi \Delta\mu L/\lambda) = \sum_{n=-\infty}^{\infty} \Phi_n \sin\{2\pi n[v^*t - (x/\lambda^*)]\}. \quad (13)$$

Since the  $\Phi_n$ 's can be determined experimentally,  $\Delta\mu$  can be computed as a function of  $x$  and  $t$ , thereby giving the waveform. This result contains the assumption that the phases between fundamental and harmonic components are either zero or  $180^\circ$ . Actually  $\Phi_n^2$  is measured, but since the approximate value is usually known, the proper sign may be assigned.

Since no measurements have been made using this method, evaluation of this method can only be speculative. It certainly appears that this method is more direct than that of ZANKEL and HIEDEMANN. However, the accuracy of measurements made by this method, as well as by the method suggested by HARGROVE, depends directly on the accuracy of light

intensity measurements, while those of ZANKEL and HIEDEMANN are based mainly on shapes of curves.

Even with recent developments, the interpretation of experimental results for obtaining detailed information about waveforms can be very tedious. Two methods using wide beams of light have been attempted which do not require tedious calculations for the determination of ultrasonic waveform: the use of a transmission plate, and diffraction by two parallel ultrasonic beams. The utilization of a transmission plate for this purpose was developed independently by MIKHAILOV and SHUTILOV [20] and by ZANKEL and HIEDEMANN [21]. Essentially the method developed by both groups is the same and, aside from the method of detection, is quite similar to a procedure previously used by ZAREMBO, KRASIL'NIKOV and SHKLOVSKAIA-KORDI [22] for determining waveforms by a non-optical method.

It is known that a thin plate placed in a sound beam will, at certain angles of incidence, pass sound of certain frequencies while reflecting other fre-

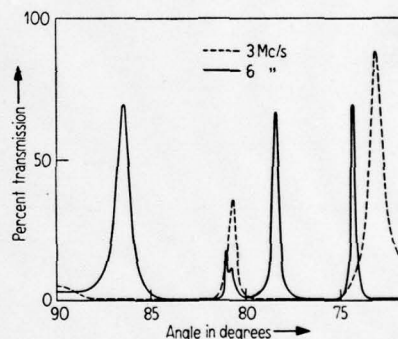


Fig. 10. The transmission coefficients of a certain steel plate in  $\text{CCl}_4$  at 3 and 6 Mc/s as a function of the angle between the plate and the sound. ZANKEL and HIEDEMANN [21].

quencies. Fig. 10 is an example of the transmission coefficients of a particular 1 mm steel plate as a function of incident angle for 3 Mc/s and 6 Mc/s ultrasonic waves in  $\text{CCl}_4$ . Starting with a 3 Mc/s sound signal, ZANKEL and HIEDEMANN [21] allowed the signal to travel far enough to become significantly distorted and then pass through the transmission plate. Fig. 11 shows the resulting diffraction patterns when light passes through the sound immediately behind the plate. Fig. 11 a is a picture taken when the plate was at an angle at which the fundamental could be transmitted. Fig. 11 b is taken at an angle at which most of the fundamental is reflected and the second harmonic was transmitted. It is seen that the spacing between diffraction orders is doubled, as would be the case when the ultrasonic frequency is doubled. By measuring the light intensity as a function of sound pressure, they observed



Fig. 11. Photographs of diffraction from the fundamental and second harmonic components of an ultrasonic wave obtained with the use of an acoustic filter. ZANKEL and HIEDEMANN [21].

that when moderate pressures were used the second harmonic component of the distorted wave was proportional (within the experimental accuracy) to the square of the sound pressure.

MIKHAILOV and SHUTILOV [20] were able to observe harmonics as high as the sixth by using transmission plates of various thicknesses placed at right angles to sound beams in water, although in this case the fundamental component transmitted was not always negligible. No quantitative measurements of waveform were made in their studies, although an increase of finite amplitude distortion with distance was indicated. Recently ADLER [23] used this method to measure distortion parameters in xylene and water.

The transmission plate method of measuring has the advantage over the other methods that the measurements are more direct, and do not depend upon detailed comparisons. It is probably the most sensitive method described here. By using such a calibrated transmission plate, ZANKEL and HIEDEMANN were able to detect much lower levels of distortion than by the other methods. For meaningful measurements of finite amplitude distortion it is necessary to avoid multiple reflections between the plate and the transducer.

The use of a transmission plate has the disadvantage of the presence of an object in the sound field; however, the measurement of the intensities of diffracted orders and the interpretation of these can be tedious if no such plate is used. ZANKEL [24] suggested a method that has the advantages of easy interpretation without the disadvantage of placement of an object in the sound field. It was predicted by LUCAS [25] that if a sinusoidal sound beam travelled parallel to and  $180^\circ$  out of phase with another identical beam, there would be no diffraction of light passing through them, i.e. the diffraction effects would cancel. ZANKEL was able to obtain near cancellation in this manner. He also showed that when the fundamental components of distorted waves were  $180^\circ$  out of phase, the even harmonics were in phase, resulting in a diffraction pattern in which the spacing is doubled. Fig. 12 a is a diffraction pattern obtained using one transducer. Fig. 12 b is that obtained using the same transducer



Fig. 12. Photographs of the diffraction spectrum produced by (a) a single distorted 2 Mc/s ultrasonic wave with about 0.6 atm of sound pressure and (b) two such waves parallel and with fundamental components  $180^\circ$  out of phase. ZANKEL [24].

nearly adjacent to another one which had its fundamental  $180^\circ$  out of phase.

ZANKEL's method has been used by MAYER and HIEDEMANN [29] to study finite amplitude distortion in more detail. They found that it was possible to demonstrate the growth of the second harmonic with distance and intensity by observing the light intensity in the zero and the second diffraction orders. The intensities of these orders for various distances from 1 Mc/s transducers as functions of the RAMAN-NATH parameter of the "cancelled" fundamentals are given in Fig. 13. From these inten-

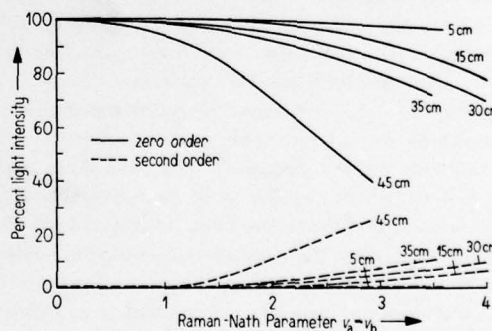


Fig. 13. Light intensity in zero order and second order vs RAMAN-NATH parameter (voltage across the transducers) for different distances. MAYER and HIEDEMANN [29].

sities it was possible to determine how the ratio of the amplitude of the second harmonic to that of the fundamental varied with distance. ZANKEL as well as MAYER and HIEDEMANN observed that the cancellation is often not complete. A possible explanation for this is that the light is no longer perpendicularly incident on the second ultrasonic beam, and therefore only under special conditions will the cancellation be complete.

Recently MERTENS [30] theoretically studied the behavior of two adjacent parallel ultrasonic waves, one of which is a harmonic of the other. He pre-

dicted conditions for which the adjacent beams would behave as if superposed. For the cancellation experiments performed these conditions are similar to those imposed by eq. (1). One must also make allowances if the waves are not quite adjacent. Another difficulty encountered in the cancellation experiments was due to inhomogeneities in the ultrasonic fields.

### 5. Studies of the broadened image

It has been stated previously that the study of broadened images by BREAZEALE [6], [7] first established a qualitative relationship between the asymmetry in diffraction patterns and finite amplitude distortion. However, the studies of the broadened image for quantitative measurements of waveform came after the successes of studies of discrete diffracted orders. The reason for this is that although diffraction using wide beams of light and sinusoidal ultrasonic waves was theoretically understood previously, a theoretical understanding of the broadening produced by sinusoidal waves and experimental verification of this has come about only recently [14].

A successful quantitative study of the broadened image for the purpose of determining waveform of finite amplitude waves has been made by HARGROVE [26]. In this study he used discrete diffraction to determine the waveform, and then compared this result with results using the broadened image. The discrete diffracted orders were used to determine the amount of second harmonic present when he held  $v=2.40$  at various distances. This was done using the method which requires only measurement of the first orders of diffraction. Then, still assuming that harmonics higher than the second could be neglected, he calculated broadened images using eq. (10) and the values of  $v$  and  $a_2$  determined from discrete diffraction. These theoretical values were compared with experimental ones. Fig. 14 contains comparisons between theoretical (circles) and experimental (lines) values so obtained for  $D=\lambda^*/2$ . If one considers that the experimental difficulties encountered include some of those using discrete diffraction and in addition require precise alignment of the slits used (to limit the light beam width and photomultiplier slit to the wave fronts) the agreement between experiment and theory are very satisfactory. As an added check, the second harmonics were also measured by the filter plate method. At distances large enough that multiple reflections between source and plate were less significant, agreement was obtained. In these experiments, the multiple reflections appeared to make the filter plate measurements the least reliable of the three used.

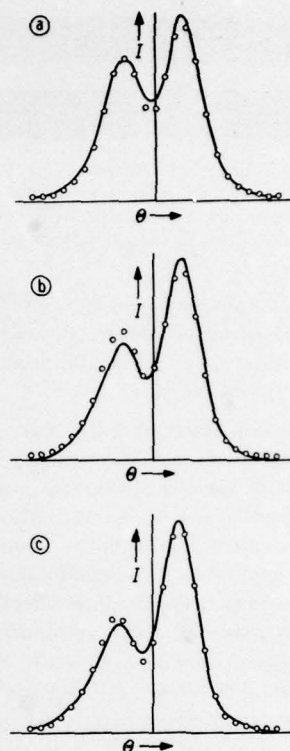


Fig. 14. Calculated (circles) and experimental (lines) light intensity  $I$  as a function of  $\theta$  for  $D=\lambda^*/2$ , for  $v=2.40$  and for (a) 5% second harmonic, (b) 10% second harmonic and (c) 15% second harmonic. HARGROVE [26].

There have been attempts to measure the light intensity at a particular portion of the broadened image (fixed  $\theta$ ) as a function of time [27], [28]. These measurements have also been attempted with stroboscopic light. Since these measurements were made previous to the derivation of eq. (5), their interpretations were essentially qualitative in nature, and therefore little will be said about these here. These methods have one common characteristic. By looking at the light as a function of time, one essentially looks at the diffraction by one small part of the wave at a given instant. Methods of this type might be more sensitive to phase differences between harmonic components than the other studies mentioned and may offer a more direct way of measuring these phase differences. Adaptations of eq. (5) which include arbitrary phase have been made by COOK [19].

### 6. Resume

An attempt has been made to discuss the relative merits of various optical methods for determining waveform. The wide light beam methods are more developed and have, therefore, been more success-

ful. The straightforward method of measuring the intensities in the diffracted orders and from this inferring the waveform has been developed to the extent that its usefulness in the study of finite amplitude distortion is established. The method of interpreting the results suggested by ZANKEL and HIEDEMANN is more complete than that suggested by MIKHAILOV and SHUTILOV, but more tedious. The one suggested by MIKHAILOV and SHUTILOV yields less information about the waveform and is limited to experiments in which a large number of diffraction orders are formed, i. e. higher ultrasonic pressures.

The filter plate method is more quickly interpreted than the method just mentioned, but has the disadvantage of placement of an object in the sound beam which affects the beam. The method utilizing parallel beams for cancellation appears difficult experimentally.

The narrow light beam methods are relatively unexplored. BREAZEAL and HIEDEMANN, and HARGROVE have given experimental evidence which indicates that studies of the broadened image can result in determinations of waveform.

#### Acknowledgment

The authors thank Dr. M. A. BREAZEAL for many critical suggestions and helpful discussions.

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ON THE DIFFRACTION OF LIGHT BY AN ULTRASONIC WAVE WITH  
ARBITRARY WAVEFORM

by  
L. E. Hargrove

Let the change in index of refraction be expressed by

$$\Delta \mu(x, t) = \sum_{j=1}^{\infty} \mu a_j \sin [2\pi(\lambda^* t - x/\lambda^*)j + \delta_j] , \quad (1)$$

where  $\mu$  is the peak change in refractive index caused by the fundamental component and  $a_j$  is the ratio of the  $j$ th harmonic component amplitude to that of the fundamental. Note the inclusion of the phase factor  $\delta_j$ ; we are not limited to ultrasonic waveforms expressible by a Fourier sine series. Assuming that the conditions for validity of the elementary Raman-Nath theory are fulfilled, the light wavefront emerging from the sound field is expressed by

$$a = \exp \left\{ \frac{-2\pi i L}{\lambda} \sum_{j=1}^{\infty} \mu a_j \sin [2\pi(\lambda^* t - x/\lambda^*)j + \delta_j] \right\} \exp (2\pi i \lambda t). \quad (2)$$

Since  $\exp \sum_k A_k = \prod_k \exp A_k$ , Eq. (2) may be written as

$$a = \prod_{j=1}^{\infty} \exp \left\{ -i v a_j \sin [2\pi(\lambda^* t - x/\lambda^*)j + \delta_j] \right\} \exp (2\pi i \lambda t) . \quad (3)$$

We now use the identity

$$\exp (i a \sin b) = \exp [ -i a \sin(-b) ] = \sum_{p=-\infty}^{+\infty} J_p(a) \exp (-i p b) \quad (4)$$

to express Eq. (3) in the form

$$a = \prod_{j=1}^{\infty} \sum_{k_j=-\infty}^{+\infty} J_{k_j}(v a_j) \exp [-2\pi i k_j(\lambda^* t - x/\lambda^*)j - i k_j \delta_j] \exp (2\pi i \lambda t). \quad (5)$$

or, by rearrangement of terms,

$$a = \prod_{j=1}^{\infty} \sum_{k_j=-\infty}^{+\infty} J_{k_j} (va_j) \exp (-ik_j \delta_j) \exp [2\pi i(\lambda - jk_j \lambda^*)t + 2\pi i j k_j x / \lambda^*] . \quad (6)$$

In eq. (6) a term for particular  $k_j$  and  $j$  represents a light component progressing in the direction satisfying

$$\sin \theta_{k_j} = -jk_j \lambda / \lambda^* \quad (7)$$

having optical frequency  $\lambda - jk_j \lambda^*$ . We may drop the time and space dependence in Eq. (6) to obtain

$$a' = \prod_{j=1}^{\infty} \sum_{k_j=-\infty}^{+\infty} J_{k_j} (va_j) \exp (-ik_j \delta_j) . \quad (8)$$

We wish to select only those terms from Eq. (8) which represent light propagating in the direction  $\sin \theta_n = -n\lambda / \lambda^*$  in order to calculate the amplitude of the  $n$ th diffraction order. Only one term in the summation for a particular value of  $j$  represents light propagating in a given direction. Let us choose a single term from each of the summations and form the indicated product. We obtain

$$\phi'_{k_1, k_2, \dots} = J_{k_1} (v) J_{k_2} (va_2) J_{k_3} (va_3) \dots \exp [-i(k_1 \delta_1 + k_2 \delta_2 + k_3 \delta_3 + \dots)] . \quad (9)$$

We must now choose the proper set of terms such as Eq. (9) which must be combined to give the amplitude of the  $n$ th order of diffraction. To select those terms representing light propagating in the direction given by  $\sin \theta_n = -n\lambda / \lambda^*$  we must require that

$$k_1 + 2k_2 + 3k_3 + \dots = n \quad (10)$$

and then sum all terms which satisfy the condition in Eq. (10).

We may proceed by allowing variation of all the  $k_j$ 's except  $k_1$  and then require that  $k_1$  be chosen to satisfy Eq. (10), that is, require that

$$k_1 = n - 2k_2 - 3k_3 - \dots \quad (11)$$

The final expression for the amplitude of light in the  $n$ th order is then

$$\begin{aligned} \phi_n = \sum_{k_2, k_3, k_4, \dots} \sum_{\dots} \sum_{\dots} \dots \sum_{\dots}^{+v} J_{n-2k_2-3k_3-4k_4-\dots}^{(v)} J_{k_2}(va_2) J_{k_3}(va_3) \\ J_{k_4}(va_4) \dots (\text{times}) \exp [-1(k_2\delta_2 + k_3\delta_3 + k_4\delta_4 + \dots)] , \end{aligned} \quad (12)$$

where  $\delta_1$ , which may be arbitrarily chosen as a reference, is chosen equal to zero. The light intensity is obtained from

$$I_n = |\phi_n|^2 \quad (13)$$

## On the Theory of the Propagation of Plane, Finite Amplitude Waves in a Dissipative Fluid

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The dissipationless theory for harmonic generation in an initially sinusoidal, plane, finite-amplitude wave is used as a basis for calculation of the harmonic components of such a wave in a fluid with dissipation. The assumption used is shown to lead approximately to the relationships of the Fox and Wallace theory. The result is given as a Fourier series, with graphs of the first few harmonic components for two specific cases. The series representation is valid for distances  $X \leq L$ , where  $L$  is the discontinuity distance for the dissipationless case.

### NOTATION

- $C'$  = Velocity of wave phase points.  
 $C_0$  = Sound velocity for infinitesimal amplitudes.  
 $P$  = Pressure.  
 $P_0$  = Internal (equilibrium) pressure.  
 $\rho$  = Density.  
 $\rho_0$  = Equilibrium density.  
 $A, B$  = Empirical constants in the equation of state.  
 $X$  = Distance from origin of wave.  
 $U$  = Particle velocity.  
 $\alpha$  = Infinitesimal amplitude absorption coefficient for the fundamental frequency component.  
 $f(n)$  = Factor by which  $\alpha$  must be multiplied in order to obtain the absorption coefficient of the  $n$ th harmonic in the medium of interest.  
 $\nu$  = Fundamental frequency.  
 $L$  = Discontinuity distance for dissipationless case.  
 $K = X/L$  = Reduced distance.  
 $\tilde{K} = 10X/L$ .  
 $P_n(K)$  = Pressure amplitude of the  $n$ th harmonic wave component, measured at reduced distance  $K$ .  
 $\delta_n(\tilde{K})$  = Harmonic generation parameter (Fox and Wallace).

### INTRODUCTION

FOX and Wallace<sup>1</sup> have presented a theory describing the propagation of plane, finite-amplitude acoustic waves in dissipative fluids. In their (large amplitude) theory, an approximate calculation of the behavior of the harmonic components of an initially sinusoidal wave was made by considering the net change of the  $n$ th harmonic pressure component in an interval  $\tilde{K}$ ,  $\tilde{K}+1$ , as<sup>2</sup>

$$P_n(\tilde{K}+1) = P_n(\tilde{K}) \exp[\delta_n(\tilde{K}) - \alpha f(n) \Delta X]. \quad (1)$$

<sup>1</sup> F. E. Fox and W. A. Wallace, J. Acoust. Soc. Am. 26, 994 (1954).

<sup>2</sup> It was subsequently assumed that each harmonic could itself undergo finite amplitude distortion, and a factor of the type  $\exp \delta_1(\tilde{K})$  must multiply each second and third harmonic equation of the type of Eq. (1). This would seem unnecessary, as graphical analysis of Eq. (4) would provide such distortion automatically; see also Eq. (7), from which the harmonic ratios are independent of amplitude.

Here  $\tilde{K}+1$  and  $\tilde{K}$  denote the distance  $X$  in terms of  $L/10$  ( $L/10 = \Delta X$ ), that is, one considers the end points of the interval from  $\tilde{K}L/10$  to  $(\tilde{K}+1)L/10$ . The  $\exp \delta_n(\tilde{K})$  factors were obtained from a graphical construction, utilizing the fact that the phase velocity of the wave is given by the local values of  $(dP/d\rho)$  and particle velocity<sup>3</sup>

$$C' = (dP/d\rho)^{1/2} + U. \quad (2)$$

Assuming an equation of state to second-order terms

$$P = P_0 + A(\rho - \rho_0/\rho_0) + \frac{1}{2}B(\rho - \rho_0/\rho_0)^2, \quad (3)$$

one obtains

$$C' = C_0 + [1 + (B/2A)]U, \quad (4)$$

which is valid to terms of the first order in  $U$ .

One may equally well use the equation of state

$$P/P_0 = (\rho/\rho_0)^\gamma, \quad (5)$$

which gives the phase velocity

$$C' = C_0 + [(\gamma+1)/2]U, \quad (6)$$

with the same degree of approximation. In this case of a gas,  $\gamma$  is, of course, given by the ratio of the specific heats; however, in the case of a liquid,  $\gamma$  may be regarded as an empirical constant equivalent to  $(B/A)+1$ , as is shown by Eqs. (4) and (6).

The neglected terms in  $U^2$  in Eq. (4) become comparable with the linear term given when  $U \approx 10^4$  cm/sec, assuming  $B/A \approx 10$ ,  $C_0 \approx 10^5$  cm/sec,  $\rho_0 \approx 1$  g/cc; this corresponds to a wave of  $\approx 1000$  atm pressure in a typical liquid. If terms in  $U^2$  must be included in Eq. (4), the equation of state (3) would probably be inadequate.

Keck and Beyer,<sup>4</sup> Fubini-Ghiron,<sup>5</sup> and Hargrove,<sup>6</sup> have given a solution of the finite amplitude problem for the dissipationless case which describes the harmonic wave structure. This solution, for the case of an initially sinusoidal wave, is given by Hargrove in the

<sup>3</sup> Lord Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1945), Vol. II, Art. 253.

<sup>4</sup> Winfield Keck and Robert T. Beyer, Phys. Fluids 3, 346 (1960).

<sup>5</sup> E. Fubini-Ghiron, Alta Frequenza 4, 530 (1935).

<sup>6</sup> Logan E. Hargrove, J. Acoust. Soc. Am. 32, 511 (1960).

form

$$P(K) = 2P_1(0) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{J_n(nK)}{nK} \times \sin 2\pi n \left( \nu t - \frac{X}{\lambda} \right). \quad (7)$$

This result allows one to write a Fourier series for the pressure components of a wave in a dissipative medium.<sup>7</sup> It should be noted that Eq. (7) is based on the use of Eq. (4), and is not valid for the case of excessively large amplitudes. Furthermore, it is not valid for  $K > 1$ , i.e.,  $X > L$ , where  $L$ , the discontinuity distance for the dissipationless case, is given by

$$L = \frac{\rho_0 C_0^3}{\pi P_1(0) \nu [(B/A) + 2]}. \quad (8)$$

At this distance, the value of  $dP/dX$  becomes infinite so that the pressure is not single valued.

It is the purpose of this paper to formulate a functional solution of the finite amplitude problem for a dissipative fluid by making use of the Fubini-Ghiron result. It will be shown that this approach, which is based on the assumption of Thuras, Jenkins, and O'Neil,<sup>8</sup> gives approximately the results of the Fox and Wallace theory for small distance or absorption, and has the advantage that the results are expressible in functions of a continuous variable.

### ANALYSIS

It is assumed that the absorptive and transfer mechanisms for the various harmonic components occur independently, as was assumed by Thuras, Jenkins, and O'Neil. Thus, the decrease of the pressure amplitude of the  $n$ th harmonic follows the usual exponential law

$$(dP_n(K)/dK)_{\text{absorp}} = -f(n)\alpha L P_n(K)_{\text{total}}. \quad (9)$$

The total space rate of change of the amplitude of a harmonic component may be taken as the sum of the rates of change due to harmonic transfer and harmonic absorption:

$$(dP_n(K)/dK)_{\text{total}} = (dP_n(K)/dK)_{\text{transfer}} - f(n)\alpha L P_n(K)_{\text{total}}. \quad (10)$$

Before solving Eq. (10), note that this linear addition leads approximately to the Fox and Wallace relations, as is seen on conversion of Eq. (10) to a finite difference relationship. For small  $K$  or  $\alpha L$ , one can assume  $(P_n(K)_{\text{transfer}}/P_n(K)_{\text{total}}) \approx 1$ , and can rewrite Eq. (10) in the form

$$d(\ln P_n(K)_{\text{total}}) = d(\ln P_n(K)_{\text{transfer}}) - \alpha f(n) dx. \quad (11)$$

In terms of the intervals proposed by Fox and Wallace, we have

$$P_n(\tilde{K}+1) = P_n(\tilde{K}) \times \exp\{\ln[P_n(K+0.1)/P_n(K)]_{\text{transfer}} - \alpha f(n)\Delta X\}, \quad (12)$$

but

$$\ln[P_n(K+0.1)/P_n(K)]_{\text{transfer}} = \delta_n(\tilde{K}), \quad (13)$$

as can be verified numerically from Eq. (7) and the graphically determined values from Fox and Wallace, taking into account the change in sign of  $\delta_1(\tilde{K})$  which they introduced.

Equation (10) may, therefore, be taken as approximately equivalent to the Fox and Wallace equations, except for the inclusion of the factor  $\exp[\delta_1(\tilde{K})]$  in their result for the second and third harmonics, as noted before.<sup>7</sup> Comparison shows, however, that  $\delta_1$  is small in comparison with  $\delta_2$  or  $\delta_3$  for all but the largest  $K$  values, so that this factor does not appreciably change the equivalence.

### SOLUTION

Integration of Eq. (10), using Eq. (7), gives an integral equation of the second kind

$$P_n(K) = \frac{2P_1(0)J_n(nK)}{nK} - f(n)\alpha L \int_0^K P_n(K') dK'. \quad (14)$$

The solution of Eq. (14) by the method of successive substitutions<sup>9</sup> (assuming  $\alpha L$  approximately constant) is an infinite alternating series for the  $n$ th harmonic amplitude of the form

$$P_n(K) = A_n(K) - B_n(K) + C_n(K) - D_n(K) + E_n(K) - \dots + \dots \quad (15)$$

The first five terms are found to be

$$A_n(K) = 2P_1(0)J_n(nK)/nK, \quad (16a)$$

$$B_n(K) = \frac{2P_1(0)f(n)\alpha L}{n^2} \times \left( \sum_{q=1}^{\infty} 2J_{n+2q}(nK) - J_n(nK) \right), \quad (16b)$$

$$C_n(K) = \frac{4P_1(0)f^2(n)\alpha^2 L^2}{n^3} \sum_{q=1}^{\infty} (2q-1)J_{n+2q-1}(nK), \quad (16c)$$

$$D_n(K) = \frac{8P_1(0)f^3(n)\alpha^3 L^3}{n^4} \sum_{q=1}^{\infty} q^2 J_{n+2q}(nK), \quad (16d)$$

$$E_n(K) = \frac{16P_1(0)f^4(n)\alpha^4 L^4}{n^5} \times \sum_{q=1}^{\infty} q^2 \sum_{r=0}^{\infty} J_{n+2(q+r)+1}(nK). \quad (16e)$$

<sup>7</sup> This possibility has also been suggested by Keck and Beyer.<sup>4</sup>  
<sup>8</sup> A. L. Thuras, R. T. Jenkins, and H. T. O'Neil, J. Acoust. Soc. Am. 6, 173 (1935).

<sup>9</sup> W. V. Lovitt, *Linear Integral Equations* (Dover Publications, New York, 1950).

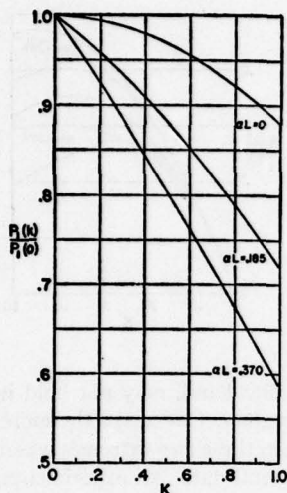


FIG. 1. Fundamental frequency component, in terms of the initial fundamental pressure.

The solution is thus expressible as the series

$$P(K) = \sum_{n=1}^{\infty} (-1)^{n+1} P_n(K) \sin 2\pi n \left( \nu t - \frac{X}{\lambda} \right), \quad (17)$$

where the  $P_n(K)$  are the harmonic amplitudes given by Eqs. (15)–(16). We must take note of the fact that, as Eq. (7) holds only for  $K \leq 1$ , Eq. (17) also holds only in this region. For constant  $\alpha L$ , the series Eqs. (15) and (16) converge absolutely and uniformly in  $K \leq 1$ .

Terms following those given in Eqs. (16) may be obtained by successively multiplying Eq. (16e) by  $-2f(n)\alpha L/n$ , adding  $2s+1$  to the order of the Bessel function, and summing over  $s$  from zero to infinity. The correction terms in Eqs. (16) are seen to cause the predicted pressure in any harmonic to be less for a given  $K$  value than that predicted by Eq. (7), as expected.

### DISCUSSION

For a given absorption law  $f(n)$  and reduced distance, the solution is evidently a function only of the product  $\alpha L$ . Graphs of Eq. (15) for the fundamental, second and third harmonic amplitudes are given in Figs. 1, 2, and 3, with the curves from Eq. (7) (dissipationless) for comparison. The  $\alpha L$  values specified for the figures are 0.185 and 0.370. This corresponds to  $P_1(0) = 1.0$  and 0.5 atm, respectively, in the case of water at a frequency of 5 Mc/sec, taking  $f(n) = n^2$  and  $(B/A) = 5$ ; this value of  $B/A$  is given by Beyer<sup>10</sup> for  $T = 20^\circ\text{C}$ . The dependence of the harmonics on distance is that which one would expect; the second and third harmonics cease growing at about the same  $K$  value and decrease slowly at larger  $K$  values, which suggests the phenomenon of "waveform stabilization."

<sup>10</sup> Robert T. Beyer, J. Acoust. Soc. Am. 32, 719 (1960).

A comparison of the results of the present theory with those of the Fox and Wallace theory has been made for the fundamental and second harmonic frequency components. The fundamental frequency component, as calculated from the Fox and Wallace theory with constant  $\Delta X$ , yields graphs which very nearly coincide with the curves of Fig. 1, the greatest discrepancy being only  $\Delta P_1(K) = 0.02$ , or 3%, at  $K = 1$  for  $\alpha L = 0.370$ . In the case of the second harmonic, calculations have been made from the Fox and Wallace theory by assuming the dissipationless value for  $P_2(0.1)$  from Eq. (7), and then calculating forward with Eq. (1) for constant  $\Delta X$ . The results for the cases  $\alpha L = 0.185$  and 0.370 are shown in Fig. 4. The Fox and Wallace curves are seen to follow the curves of Fig. 2 up to  $K$  values of about 0.6 and 0.4, respectively, and then lie below them, but maintain the same general shape. The discrepancies between the Fox and Wallace theory and the present theory increase with  $K$  and  $\alpha L$ , as expected from the approximation in Eq. (11) relating the two theories.

The perturbation analysis of Keck and Beyer also yields functions of the product  $\alpha L$  and of the reduced distance  $K$ . A calculation of the second harmonic from their result for the cases  $\alpha L = 0.185$  and 0.370 is shown in Fig. 4. It is evident that there is fairly good agreement between the present calculation and their results, with the discrepancy increasing as  $\alpha L$  increases.

There remains some question concerning the exact interpretation of the reduced variable  $K$ . For the dissipationless case, the value of  $X$  corresponding to a given  $K$  value is computed on the basis of the initial fundamental pressure. It might equally well be regarded as based on a fundamental pressure component which varies with distance in the manner predicted by Eq. (7). On this interpretation, one may expect the  $K(X)$  relationship to depart from linearity as the fundamental frequency component of the pressure is absorbed, and

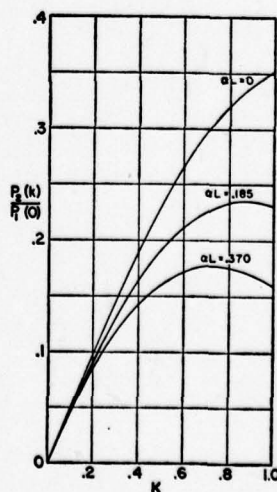
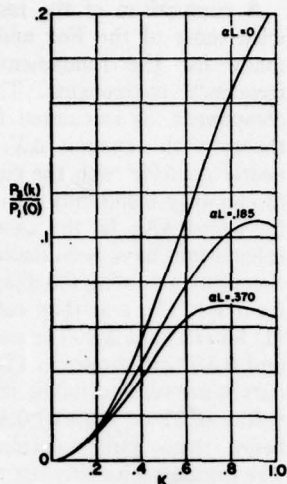


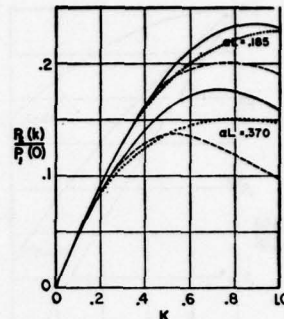
FIG. 2. Second harmonic frequency component, in terms of the initial fundamental pressure.

FIG. 3. Third harmonic frequency component, in terms of the initial fundamental pressure.



the degree of departure could be estimated numerically. The dependence of the harmonic pressure on distance would then become a more complicated function than Eq. (17), as the value of  $L$  to be used in Eq. (16) would be regarded as a function of the distance. In order to avoid difficulties of this sort, the first experimental verification should be made for small  $\alpha L$  values (large pressure), with  $L$  treated as constant as an approximation.

FIG. 4. Second harmonic as predicted by Eq. (1) (dashed line), as predicted by Eq. (15) (solid line), and as predicted by Keck and Beyer (dotted line).



Equation (10), on the other hand, may not hold in the case of very small  $L$  values. One may, therefore, hope to find a region between these two extremes where Eq. (17) describes experimental data. An experimental investigation is planned in the near future.

#### ACKNOWLEDGMENT

The author wishes to express his appreciation to Professor E. A. Hiedemann for his interest in this work, to Dr. L. Hargrove for suggesting Eq. (13) and to Mr. L. Fenton for assistance in numerical calculating. This work was supported by the Office of Naval Research.

Optical Method for Ultrasonic Velocity  
Measurements at Liquid-Solid Boundaries

by

Walter G. Mayer and John F. Kelsey

Abstract. An optical method is used to measure the energy ratio of reflected and incident ultrasonic waves at a liquid-solid interface. The ultrasonic velocities in the solid are calculated from the angles of maximum reflection in the liquid.

The intensity ratio of reflected to incident ultrasonic waves at a liquid-solid boundary as a function of angle of incidence is given by Ergin<sup>1</sup> as

$$\left(\frac{R}{I}\right)^2 = \frac{[\cos \beta - A \cos \alpha (1-B)]^2}{[\cos \beta + A \cos \alpha (1-B)]^2} \quad (1)$$

where  $\alpha$  is the angle of incidence in the liquid measured from a line normal to the interface,  $\beta$  and  $\gamma$  are the angles of refraction of the longitudinal and shear wave in the solid. The quantities A and B are defined by

$$A = \rho_2 V_L / \rho_1 V_I, \quad (2)$$

$$B = 2 \sin \gamma \sin 2\alpha [\cos \gamma - (V_L/V_S) \cos \beta], \quad (3)$$

where  $\rho_1$  and  $\rho_2$  are the densities of the liquid and the solid, respectively, and  $V_I$  is the velocity of the incident wave in the liquid;  $V_L$  and  $V_S$  are the velocities of the refracted longitudinal and shear waves in the solid.

Substituting accepted values for the densities and velocities of water and Plexiglas in eq. (1) one obtains the curve shown in Fig. 1a for the intensity ratio  $(R/I)^2$ . Figure 1b shows this ratio for a water-aluminum boundary. The ultrasonic wave is incident in the water in both cases. In order to obtain these curves one has to use the appropriate angles  $\beta$  and  $\gamma$  for a given angle of incidence  $\alpha$ . These angles are found from Snell's law

$$\begin{aligned} V_I/V_L &= \sin \alpha / \sin \beta, \\ V_I/V_S &= \sin \alpha / \sin \gamma. \end{aligned} \quad (4)$$

It can be seen from eq. (4) that  $\sin \alpha = V_I/V_L$  or  $\sin \alpha = V_I/V_S$  at the critical angles for the longitudinal and shear wave where  $\sin \beta$  or  $\sin \gamma$  equal unity. One can obtain  $V_L$  and  $V_S$  if  $V_I$  is known provided the critical angles can be located. Equation (1) and Fig. 1 show that the ratio  $(R/I)^2 = 1$  at the critical angles. The associated peaks in the intensity of the reflected wave can be located experimentally and can be used to calculate  $V_L$  and  $V_S$  for the solid<sup>2</sup>.

An optical method is used to find the angle of incidence at which the intensities of the reflected and incident beams are equal. The arrangement is shown in Fig. 2. The solid sample and the transducer are placed in a tank filled with water. While the angle of incidence is changed by rotating the transducer the sample is also rotated in such a manner that the reflected sound beam remains at right angles to the collimated light beam. The reflected ultrasonic wave produces a diffraction pattern in the plane of the photomultiplier. The light intensity in the  $n$ th order of the diffraction pattern is given by

$$I_n = J_n^2(v), \quad (5)$$

where  $v$  is proportional to the amplitude of the ultrasonic wave

producing the diffraction pattern. Keeping the output of the transducer constant and measuring the light intensity in the zero order one finds a pronounced dip at that angle of incidence where the reflected ultrasonic wave is most intense. Figure 3a shows the zero order light intensity for a 8 Mc continuous ultrasonic wave reflected from a water-Plexiglas boundary. For convenience the intensity of the incident wave is kept low enough so that the zero order Bessel function is positive for all possible values of  $v$  of the reflected wave. The critical angle is located at  $33.5^\circ$  from which one finds  $V_L = 2700$  m/sec, using eq. (4). The velocity in water,  $V_I$ , can be determined by measuring the spacing between the lines of the diffraction pattern produced by the reflected wave in the liquid. Figure 3b shows the intensity of the reflected wave obtained from the data given in Fig. 3a. The theoretical curve predicted by eq. (1) is also given.

The same technique is used to measure  $V_S$  if  $V_S > V_I$ . In this case the light intensity in the zero order reaches a minimum at the critical angle for the shear wave and remains at that level. An example is given in Fig. 4a which shows the measured light intensity in the zero order produced by a wave reflected from a water-glass boundary at angles in the vicinity of the critical angle for the shear wave. The corresponding intensity of the reflected wave is shown in Fig. 4b. The critical angle for the shear wave is  $25.4^\circ$  corresponding to a shear wave velocity of 3445 m/sec.

It should be noted that this analysis does not include surface waves or plate transmission phenomena. The method given here has the advantage that the velocity of the longitudinal and shear wave in the solid can be calculated without having to observe the waves in the solid directly.

References:

1. K. Ergin, Bull. Seism. Soc. Am. 42, 349 (1952).
2. W. G. Mayer, J. Acoust. Soc. Am. 32, 1213 (1960).

List of Figures:

Fig. 1. Intensity ratio of reflected to incident wave as a function of angle of incidence for (a) a water-Plexiglas boundary where  $V_S < V_I < V_L$  and (b) a water-aluminum boundary where  $V_I < V_S < V_L$ .

Fig. 2. Diagram of the optical arrangement.

Fig. 3. Critical angle for longitudinal wave in Plexiglas. (a) zero order light intensity in diffraction pattern produced by reflected wave in water. (b) corresponding values of  $(R/I)^2$ . Solid line shows theoretical values.

Fig. 4. Critical angle for shear wave in glass. (a) zero order light intensity in diffraction pattern produced by reflected wave in water. (b) corresponding values of  $(R/I)^2$ . Solid line shows theoretical values.

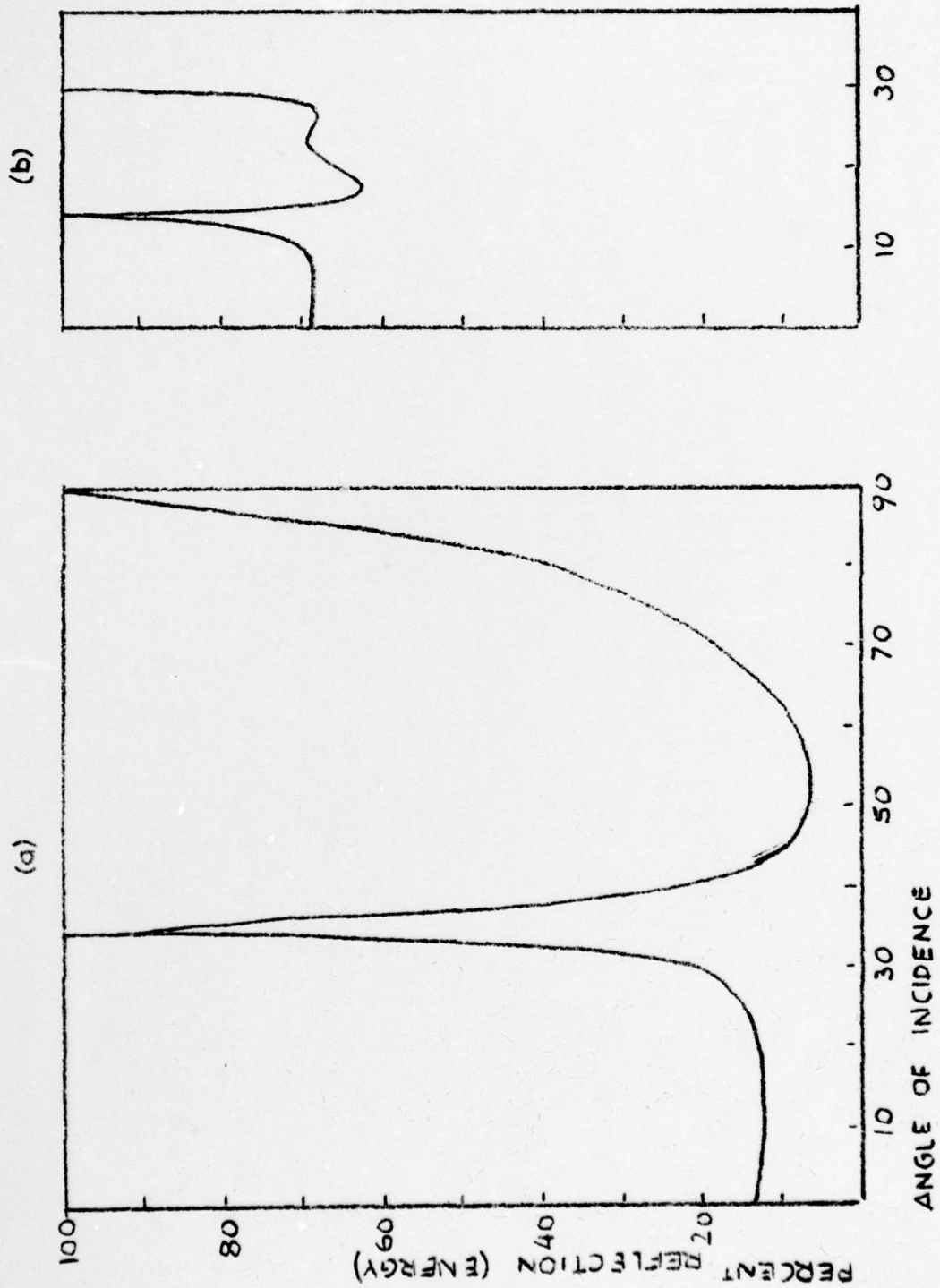


Figure 1

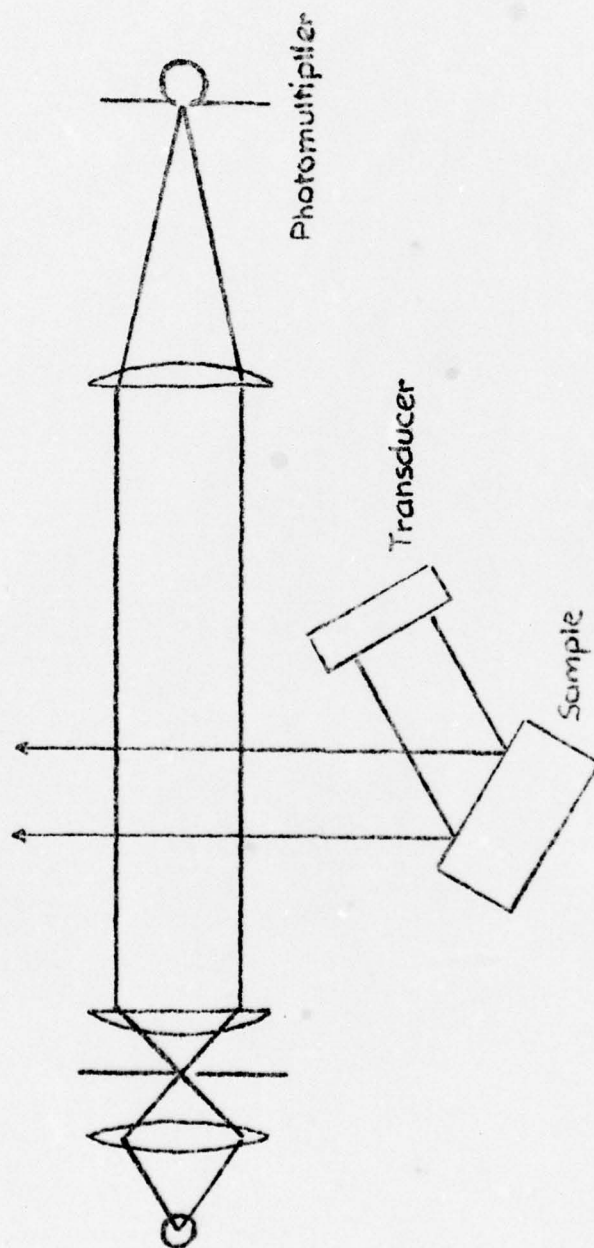


Figure 2

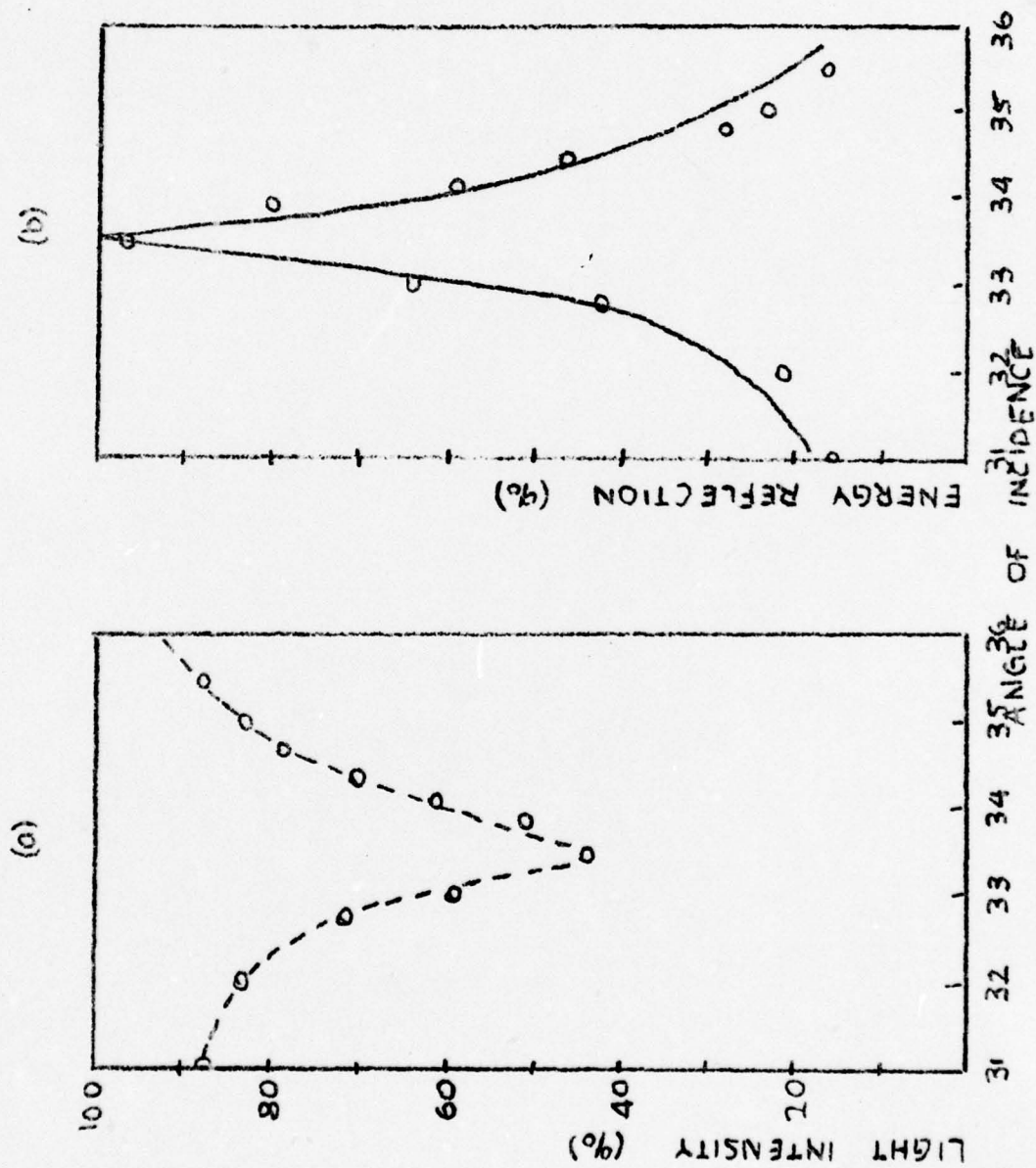


Figure 3.

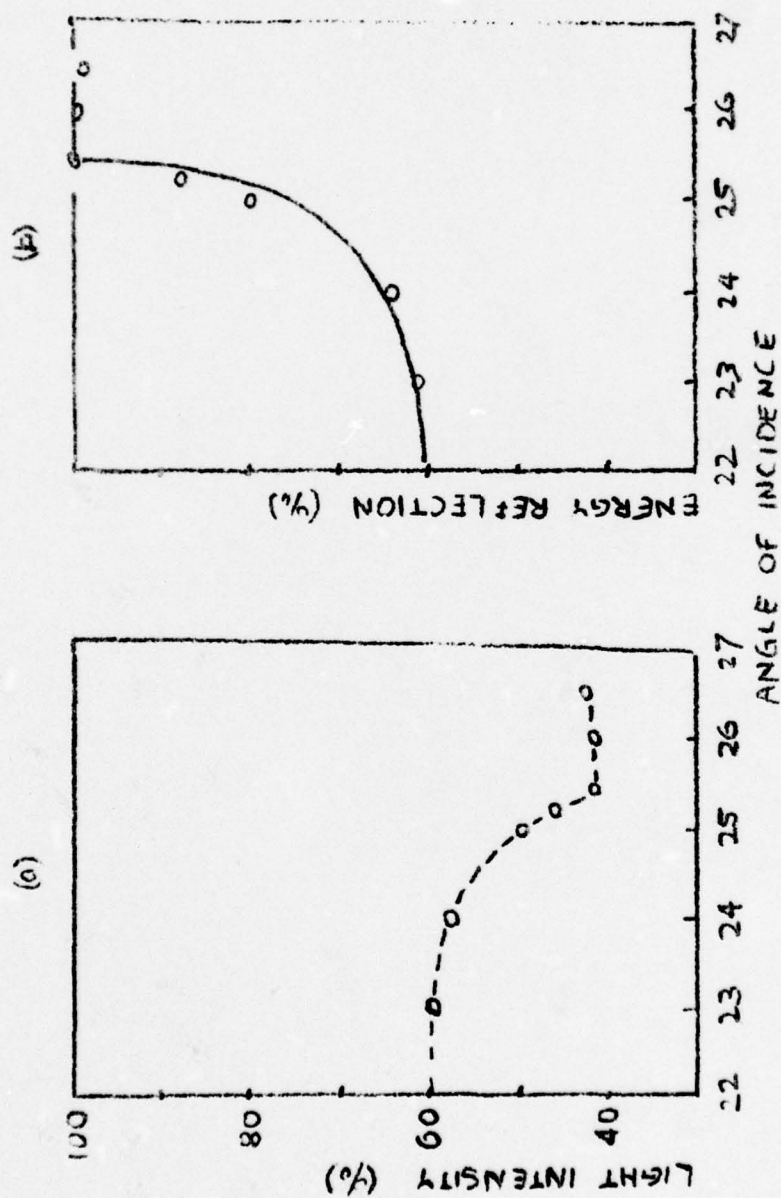


Figure 4.

Notes on Reflection from Liquid-Solid  
Interfaces (Energy Relations)

by

Walter G. Mayer

If a plane longitudinal wave impinges on a liquid-solid boundary the wave is reflected and refracted at angles which are defined in Fig. 1. For incidence in the liquid the ratio of the energy of reflected to incident wave as a function of the angle of incidence is given by

$$\left(\frac{R}{I}\right)^2 = \left[ \frac{\cos^2 \beta - A \cos \alpha (1-B)}{\cos^2 \beta + A \cos \alpha (1-B)} \right]^2, \quad (1A).$$

where

$$A = \rho_2 V_L / \rho_1 V_1, \quad (2A)$$

$$B = 2 \sin \chi \sin 2\chi [\cos \chi - (V_S/V_L) \cos \beta]. \quad (3A)$$

Snell's law is valid, i.e.

$$\frac{V_I}{V_L} = \frac{\sin \alpha}{\sin \beta}, \quad \frac{V_2}{V_S} = \frac{\sin \alpha}{\sin \chi}. \quad (4A)$$

Supposing that all the possible velocities are known and one plots  $(R/I)^2$  from eq. (1A) one obtains graphs similar to Fig. 2A, provided  $V_I < V_S < V_L$ . Four points or regions are of special interest:

(1) At  $\alpha = 0$  the reflection formula (1A) reduces to

$$\left(\frac{R}{I}\right)^2 = \left[ \frac{1-A}{1+A} \right]^2 = \left[ \frac{\rho_1 V_I - \rho_2 V_L}{\rho_1 V_I + \rho_2 V_L} \right]^2. \quad (5A)$$

This is the well-known reflection formula for normal incidence.

(2) At critical angle  $\alpha$  where longitudinal wave in solid cuts off, i.e.  $\beta = 90^\circ$ . (Point L in Fig. 2A).

(3) Beyond critical angle for longitudinal wave where according to eq. (4A)  $\sin \beta > 1$ . (Points L to S in Fig. 2A).

(4) At critical angle  $\alpha$  where shear wave in solid cuts off, i.e.  $\sin \gamma = 1$ . (Point S in Fig. 2A).

\*\*\*\*\*

To (1) : No shear wave is present in solid.

To (2) : With  $\beta = 90^\circ$ ,  $\cos \beta = 0$  in eq. (1A) and  $(R/I)^2 = 1$ . This means there is total reflection, no shear wave present in solid despite the fact that  $\sin \gamma$  is still real.

To (3) : From eq. (4A) find that  $\sin \beta > 1$ . To find  $(R/I)^2$  for this region one can use the following method:  
Rewrite eq. (1A)

$$\left(\frac{R}{I}\right)^2 = \left[ \frac{\cos^3 \beta (1 - 2AD \cos \alpha \sin \gamma \sin 2\gamma) - A \cos \alpha (1 - 2 \cos \gamma \sin \gamma \sin 2\gamma)}{\cos^3 \beta (1 + 2AD \cos \alpha \sin \gamma \sin 2\gamma) + A \cos \alpha (1 - 2 \cos \gamma \sin \gamma \sin 2\gamma)} \right]^2 \quad (6A)$$

with A given by eq. (2A) and

$$AD = \rho_2 V_S / \rho_1 V_I.$$

To find  $\cos \beta$  in cases where  $\sin \beta > 1$ , one can use (with  $\theta_1 = \frac{\pi}{2}$ )

$$\sin(\theta_1 + i\theta_2) = \sin \theta_1 \cosh \theta_2 + i \cos \theta_1 \sinh \theta_2 =$$

$$1 \times \cosh \theta_2 + i(0).$$

Example: Supposing that from Snell's law eq. (4A),  
 $\sin \beta = 1.0095$ , one therefore takes  $\cosh \theta_2 = 1.0095$ . From tables one finds that if  $\cosh = 1.0095$  the corresponding  $\sinh = 0.1375$ , and from trigonometric identities this means that  $\cos \beta = 0.1375i$ .

One has a complex quantity in eq. (6A), which is solved by taking the complex conjugate.

Then one has essentially

$$\left(\frac{R}{I}\right)^2 = \left[\frac{1a + b}{Re}\right]^2$$

Now use  $\frac{a^2}{Re} + \frac{b^2}{Re}$  as result for  $(R/I)^2$  in region L to S.

(4) At this point  $\cos \beta$  is imaginary. Also  $\gamma = 90^\circ$ , therefore,  $\sin \gamma = 1$  and  $\sin 2\gamma = 0$ . This reduces eq. (1A) to

$$\left(\frac{R}{I}\right)^2 = \left[\frac{\cos \beta - A \cos \alpha}{\cos \beta + A \cos \alpha}\right]^2$$

or, in general terms,

$$\begin{aligned} \left(\frac{R}{I}\right)^2 &= \left[\frac{1a - b}{1a + b}\right]^2 \\ &= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2iab}{a^2 + b^2}\right)^2 \end{aligned}$$

Then the absolute value of  $(R/I)^2$

$$|R/I|^2 = \frac{(a^2 - b^2)^2 + 4a^2b^2}{(a^2 + b^2)^2} = \left(\frac{a^2 + b^2}{a^2 + b^2}\right)^2 = 1.$$

Therefore, total reflection of incident energy at the critical angle for the shear wave. No energy present in solid.

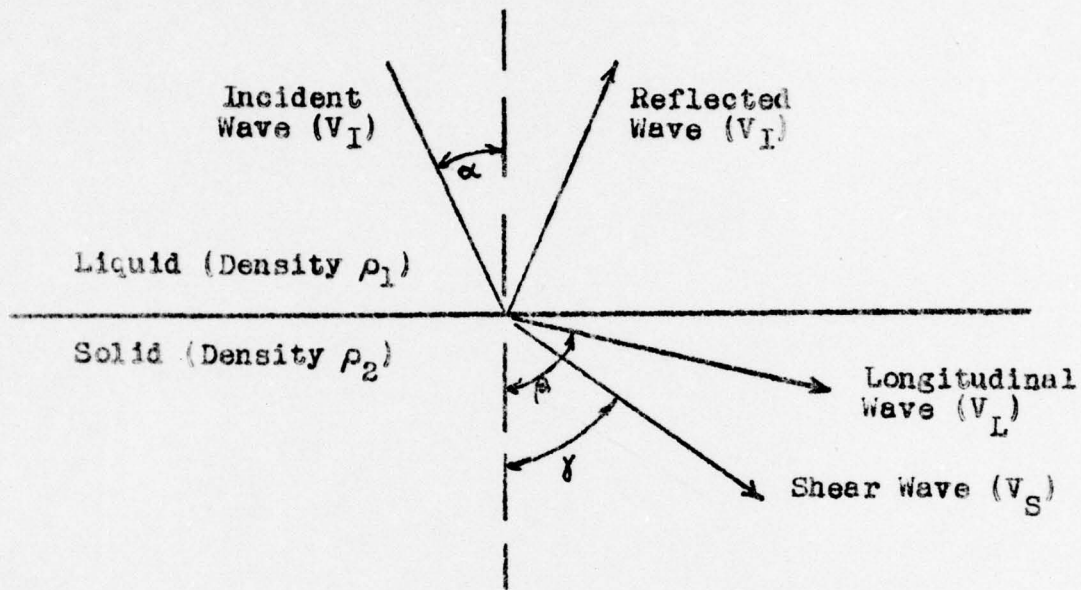
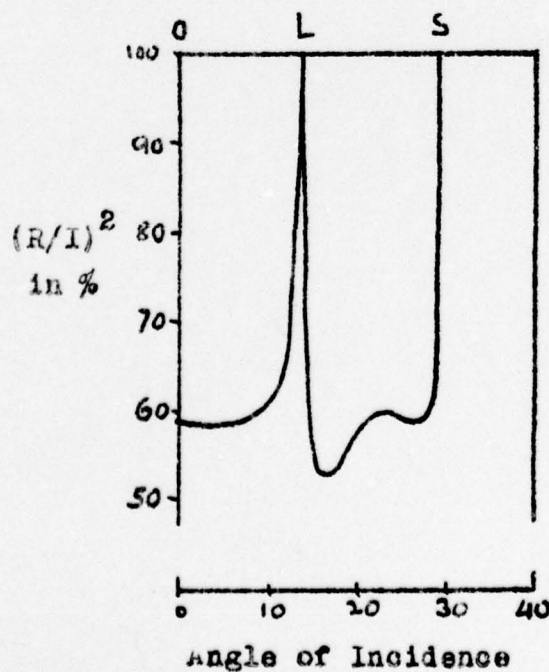


Figure 1A. Reflection and Refraction.



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Figure 2A. Reflection from an Aluminum-Water Boundary.

Measurement of the Sound Pressure Amplitude  
in Transparent Solids

by

K. Achyuthan and M. A. Breazeale

When a sound wave propagates through a substance it produces periodic variations in the density. In transparent substances, associated with these density variations are variations in the index of refraction. Therefore, one of the methods for determining the amplitude of the pressure wave is to measure the amplitude of the index of refraction variation. This can be done by observing how light is diffracted when it passes through the sound beam under consideration. If this sound beam is passing through a liquid, then relatively straightforward measurements can give one a value for the sound amplitude. If, on the other hand, the sound beam is passing through a solid the situation can be more complicated. This complication is attributable to the fact that solids exhibit stress birefringence. Thus, the state of polarization of the light must also be considered. What we would like to point out is that if one does consider the polarization of the light he can find special conditions under which the theory for the diffraction of light by ultrasonic waves in solids is almost as simple as that in liquids, and thus, it is possible to use the same procedure for measuring the sound amplitude in solids as in liquids. One difference is that the frequency range for the validity of the simplified diffraction theory of Raman and Nath is

higher in solids than in liquids because of the higher sound velocity. For example, we will be considering measurements made in glass at 10 megacycles. For various reasons the diffraction theory becomes complicated at this frequency for liquids.

The optical arrangement is shown in Figure 1. The arrangement is the same as that used with liquids except for the use of the polarizer and the wollaston prism. The wollaston prism splits the light into two beams, one of which is polarized parallel to the sound wave fronts, the other perpendicular to them. It turns out that with light polarized parallel to the sound wave fronts or perpendicular to them, there is no rotation of the plane of polarization. The simplification introduced by satisfying these polarization requirements can be seen:

$$S_w R = 1$$

$$\text{For odd orders } m = 2S + 1$$

$$I_m = 2 \sum_r \left[ J_{s-r}(V_1) J_{s+r+1}(V_1) \cos^2(\theta - \alpha) + J_{s-r}(V_2) J_{s+r+1}(V_2) \sin^2(\theta - \alpha) \right]^2$$

With polarized light

$$I_m(\parallel) = 2 \sum_r \left[ J_{s-r}(V_1) J_{s+r+1}(V_1) \right]^2$$

$$\alpha = 0$$

$$\theta = 0$$

$$I_m(\perp) = 2 \sum_r \left[ J_{s-r}(V_2) J_{s+r+1}(V_2) \right]^2$$

$$\alpha = \pi/2$$

$$\theta = 0$$

If the incident light is polarized at an arbitrary angle  $\alpha$ , then the diffracted light is in general rotated through an angle  $\theta$ . Under this condition the intensity of light in the odd diffraction orders is given by the first expression, where the J's are Bessel functions. As indicated, this expression is valid for a standing wave ratio of unity. This condition is fulfilled in the experimental situation since the wave is for all practical purposes 100 per cent reflected at the surface of the glass.

This general expression can be used to calculate the diffracted light for an arbitrary angle of polarization. If, however, the angle of polarization is either zero or  $90^\circ$ ; i.e., if the polarization is either parallel to or perpendicular to the acoustic wavefronts, then there is no rotation of the plane of polarization, and the simpler expressions can be used.

In our measurements we chose to measure the light in the first order of the diffraction pattern whose plane of polarization is parallel to the wave fronts. Then, taking the expression which is second from the bottom we find that it is necessary to evaluate only two terms for an accuracy of better than 1 per cent, which is the accuracy of the photomultiplier. These two terms are:

$$I_1 (11) = 2 \left[ J_0^2(v_1) J_1^2(v_1) + J_1^2 v_1 J_2^2(v_1) \right]$$

$$v = \frac{2\pi \mu_1 L}{\lambda} = \frac{2\pi \mu_0^2 L}{\rho c^2 \lambda} \quad qP$$

$\mu_1$  = amplitude of Index of refraction change

$\mu_0$  = Index of refraction

L = Width of Sound beam

$\lambda$  = light wavelength

$\rho$  = density

c = Sound velocity

q = strain optical constant

P = sound pressure amplitude

As can be seen, the argument  $V$  of the Bessel functions is a linear function of the pressure. Therefore, determining the light intensity at some convenient reference, for example, at a particular voltage on the quartz transducer, one finds on the curve given by the product of the Bessel functions the corresponding value of  $V$ . From this the pressure amplitude  $P$  can be determined.

In the expression for the relationship given between  $V$  and  $P$  all the quantities can be determined in a straightforward way except  $q$ , the photoelastic constant. This quantity was determined by use of the results of both ultrasonic and static measurements.

For glasses there are only two photoelastic constants. In order to determine  $q$ , one measures first the ratio of these constants by use of the ultrasonic technique of Mueller. This ratio is related to the intensity of light in a diffraction pattern when the plane of polarization is parallel to the direction of propagation of the ultrasonic wave relative to that when it is polarized perpendicular to it. The use of the wollaston prism simplifies these measurements.

The second measurement is made by use of a Babinet compensator. One measures the stress birefringence, which is a measure of the difference between the photoelastic constants. From the measured ratio and difference the value of  $q$  can be determined. Thus, we have all the quantities necessary to relate the pressure amplitude to the  $V$ -value.

We have made measurements on two glass samples in order to determine the range of applicability of the method. The samples were excited near resonance by a 10 mc quartz transducer. The light intensity in the first diffraction order was measured and was plotted as a function of quartz voltage. Superimposed on this curve was plotted the function

$I_1 = J_0^2(v) J_1^2(v)$ . The results for one sample are shown in Fig. 2. Measurements were made on four different occasions in order to determine the reproducibility of the measurements. The scatter of the points is an indication of this.

The reproducibility of the measurements is affected by two factors: temperature increase and frequency drift. An increase in temperature in the sample changes the sound velocity, and hence the wavelength. This in turn affects the standing wave pattern. Frequency drift in the oscillator has the same effect. This, of course, can be diminished by use of a more stable oscillator.

In the measurements it was observed that a greater reproducibility and greater accuracy was obtained when the frequency was not exactly on resonance. This is understandable since at resonance the amplitude is so highly dependent on frequency. By getting off the steep portion of the resonance curve accuracy of the measurements was increased, yet the acoustic energy transmitted into the sample was still great enough to be measured.

In Fig. 3 is shown a calibration curve for the particular sample used. This curve was obtained from the previous one by using the scaling factor between quartz volts and  $v$ -value. The points are the same experimental points plotted with the different ordinate. Since the value of  $V$  is proportional to the sound pressure amplitude, this quantity is also indicated on the ordinate. In this particular sample the energy density at a  $v$ -value of 0.5 is of the order of 4 ergs/cm<sup>3</sup>.

The scatter of the data gives an indication of the random error involved in measuring the quartz voltage and in keeping the oscillator tuned to the exact frequency. This error and the systematic error introduced in the determination of the value of the photoelastic constants constitute the two main sources of inaccuracy. It is to be emphasized

that the accuracy of the measurement is dependent of the sample used, the ratio of the photoelastic constants being the determining factor. If this ratio is large, then the accuracy in determining  $q$  is greater, and hence the accuracy in determining the pressure amplitude is greater. For this sample the value of the ratio of the photoelastic constants was 1.11, a fairly low value. For these measurements therefore, including both random and systematic error, an accuracy of about 30 per cent is to be expected.

On another sample the accuracy was greater because of a larger value of the ratio of the photoelastic constants. With a value of this ratio of 1.36, the overall accuracy turned out to be 16 per cent. Measurements of the light intensity as a function of quartz voltage are given in Fig. 4. These measurements were made with the sound on for very short intervals to avoid heating the sample and causing the standing wave pattern to change. This procedure has improved the scatter of the experimental points. Nevertheless, at high quartz voltages the experimental points do fall below the theoretical curve because of heating. This can be seen better in Figure 5 which shows the pressure amplitude plotted as a function of quartz voltage. At high voltages the experimental points tend to drop below the straight line. This actually means that at high voltages the standing wave pattern is different from that at low voltages, so one should expect only an approximately linear relationship. It also points out the sensitivity of the method to small changes in the standing wave pattern: ( $V = 0.2$  corresponds to energy density of  $3.2 \text{ ergs/cm}^3$ .) This, then, is the way to measure the sound amplitude in solids. With the supplementary Babinet compensator measurements it is an absolute method. Without them it gives relative values.

In conclusion, it may be observed that this method can be used to measure the sound amplitude in crystals. In such a case, however, it is necessary to use the correct pair of photoelastic constants, since for crystals one must use more than two constants to describe the photoelastic behavior.

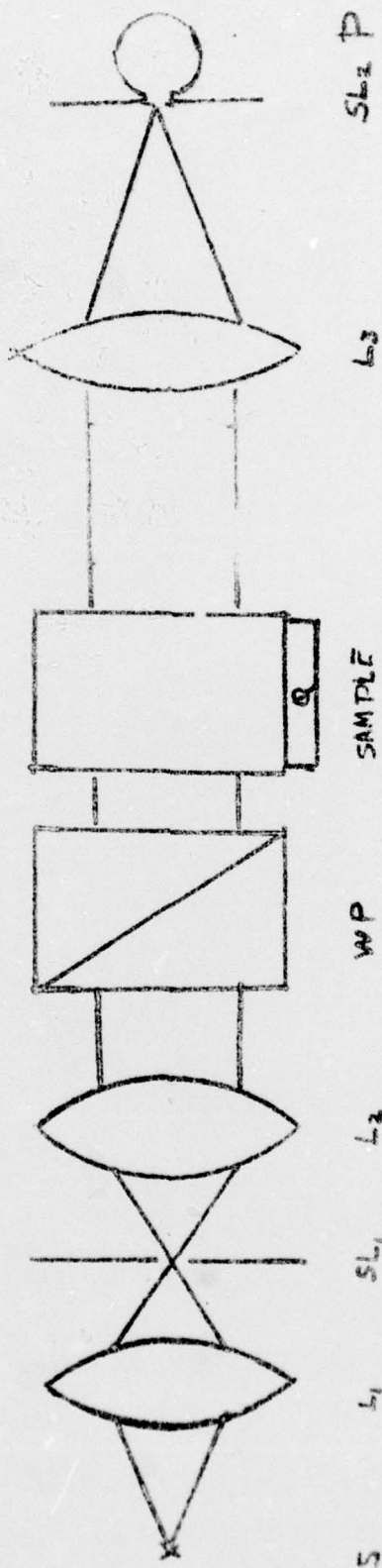
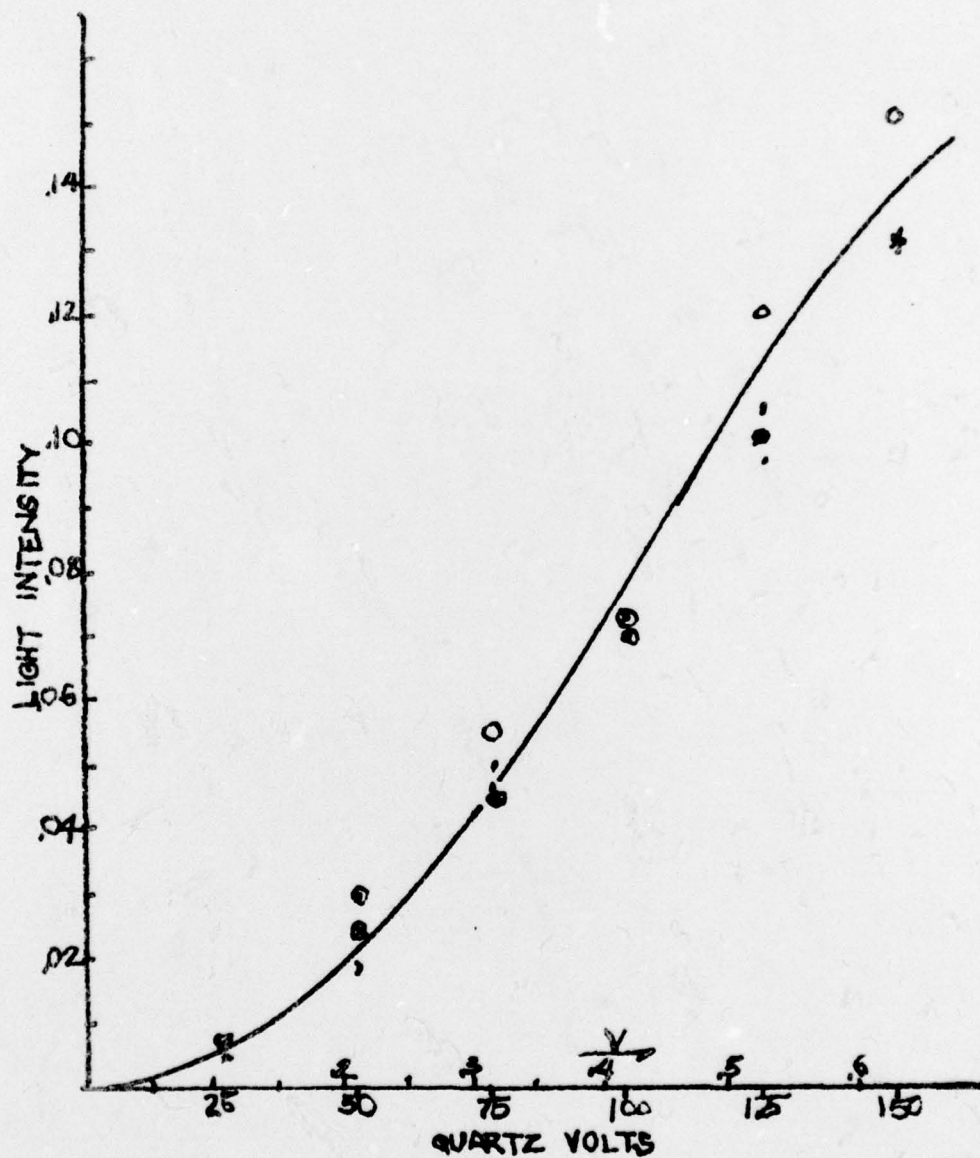
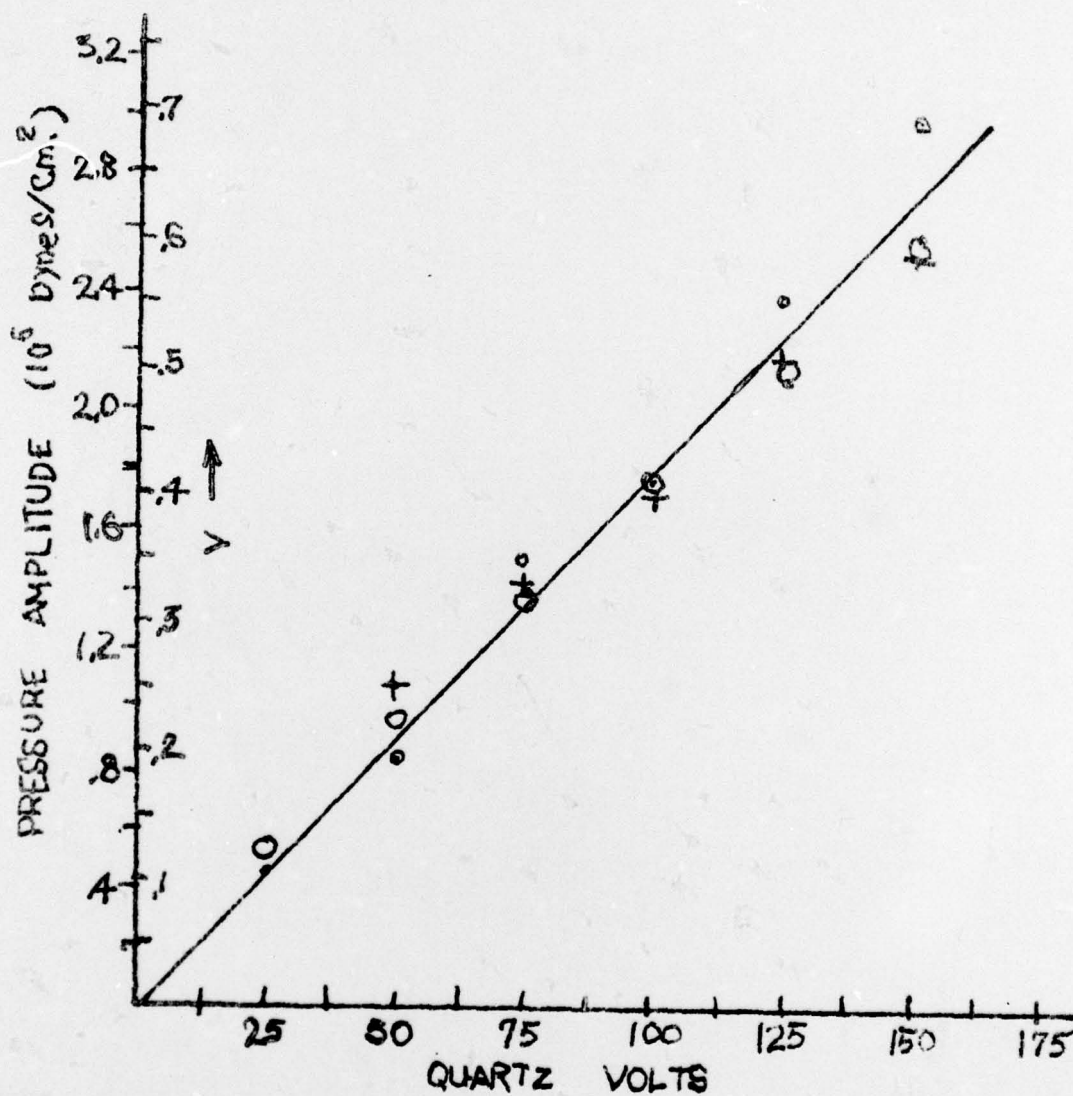


FIGURE 1. OPTICAL ARRANGEMENT



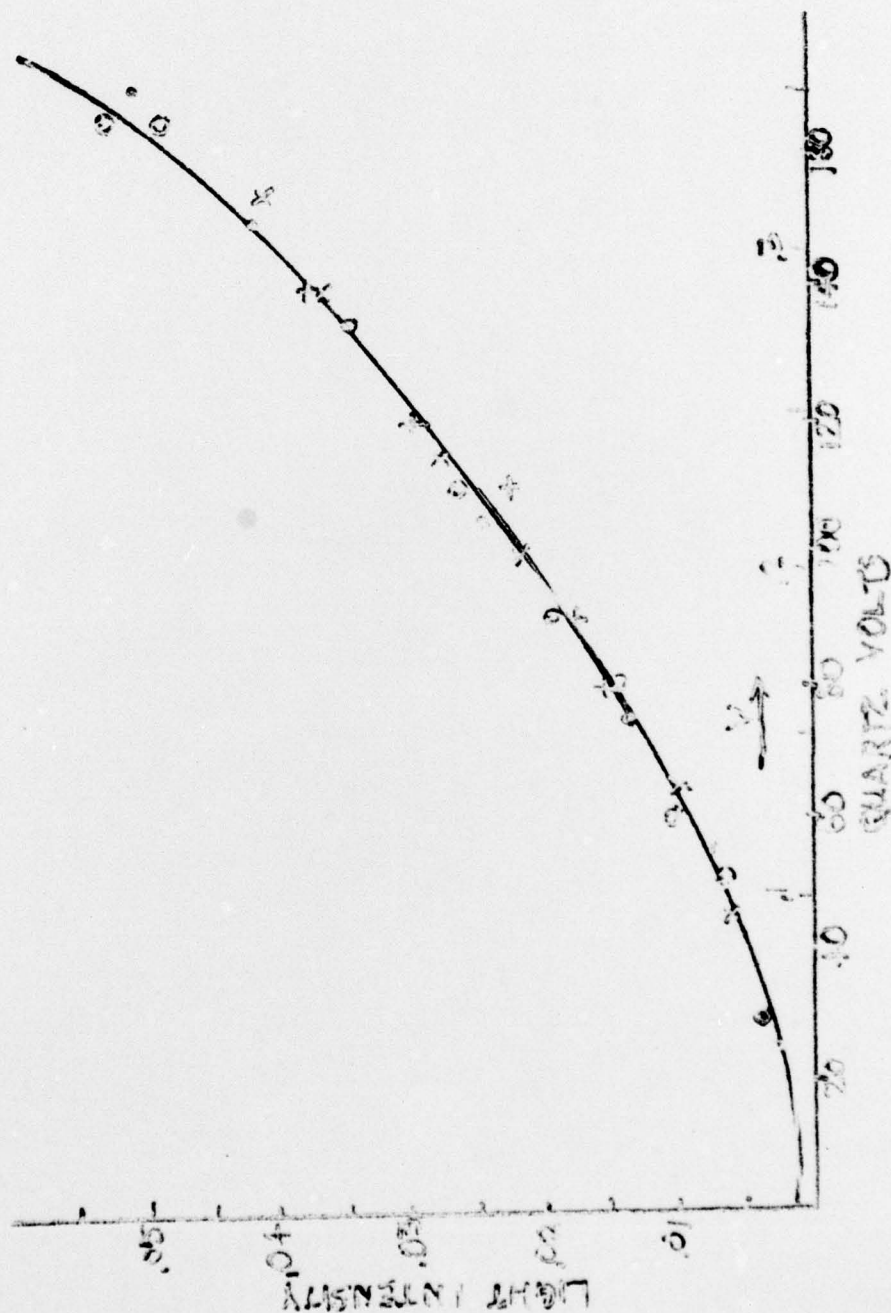
Sample no. 25

Figure 2



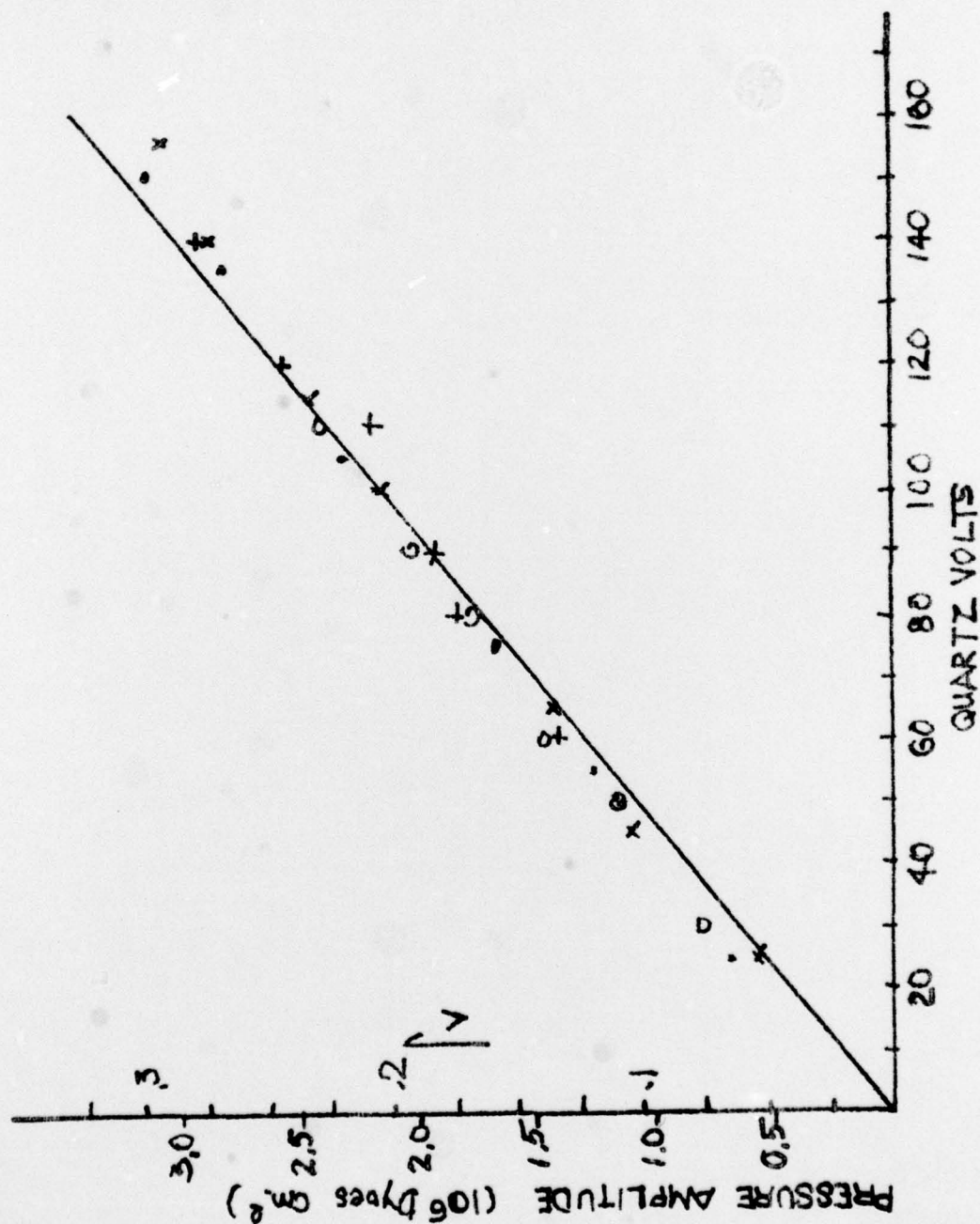
Calibration of sample no, 25, 9.82 Mc,

Figure 3



SAMPLE NO. 25, 969 M.

Figure 4



Calibration of sample no. 23 36.9 Mc.

Figure 5

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